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TO A

TREATISE ON SURVEYING

BY

SAMUEL ALSOP.
AUTHOR OF "A TREATISE ON ALGEBRA," ETC.



PHILADELPHIA:
E. C. & J. BIDDLE, No. 508 MINOR STREET,
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1857.

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KEY

TO

ALSOP'S SURVEYING.

CHAPTER III.

PLANE TRIGONOMETRY.

Article 137.

Ex. 3. Here, (Rule 3,)

As AB	59.47	A. C. 8.225702
: BC	48.52	1.685921
:: rad.		10.000000
: tan. A	39° 12′ 36″	9.911623

And (Rule 2)

As cos. A	39° 12′ 36″	A. C. 0.110791
: rad.		10.000000
:: AB		1.774298
: AC	76.75	1.885089

Ex. 4. Here, (Rule 1,)

As AC	97.23	A. C. 8.012200
: BC	75.87	1.880070
:: rad.		10.000000
: sin. A	51° 17′ 22″	9.892270

And (Rule 2)

As rad.		A. C. 0.000000
: cos. A	51° 17′ 22″	9.796148
:: AC		1.987800
: AB	60.81	1.783948

Ex. 5. Here, (Rule 3,)

As tan. A	42° 19′ 24″	A. C. 0.040636
: rad.		10.000000
:: BC	25.54	1.407221
: AB	28.045	1.447857

And (Rule 1)

As sin. A		A. C. 0.171783
: rad.		10.000000
:: BC		1.407221
: AC	37.932	1.579004

Ex. 6. Here A = 90° − C = 17° 17′ 51″

Rule 1.

As rad.		A. C. 0.000000
: sin. A	17° 17′ 51″	9.473243
:: AC	495	2.694605
: BC	147.18	2.167848

And (Rule 2)

As rad.		A. C. 0.000000
: cos. A		9.979900
:: AC		2.694605
: AB	472.612	2.674505

Ex. 7. Rule 3.

As rad.		A. C. 0.000000
: tan. A	42° 8′ 45″	9.956659
:: AB	63.2	1.800717
: BC	57.20	1.757376

And (Rule 2)

As cos. A		A. C. 0.129925
: rad.		10.000000
:: AB		1.800717
: AC	85.24	1.930642

Article 138.

Ex. 2. $AC = \sqrt{AB^2 + AC^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$

Ex. 3. $BC = \sqrt{(AC + AB).(AC - AB)} = \sqrt{77.37 \times 18.47}$
$= \sqrt{1629.0239} = 37.8.$

Ex. 4.	49.27 + 37.42 = 86.69	log. 1.937969
	49.27 − 37.42 = 11.85	" 1.073718
		2) 3.011687
	Ans. 32.05	1.505843

Article 139.

Ex. 3. Here, (Rule 1,)

As AB	37.25	A. C. 8.428874
: AC	42.59	1.629308
:: sin. C	57° 29′ 15″	9.925969
: sin. B	74° 36′ 53″	9.984151
A	47° 53′ 52″	

And (Rule 2)

As sin. C		A. C. 0.074031
: sin. A		9.870375
:: AB		1.571126
: BC	32.774	1.515532

Ex. 4. Here C = 180 − A + B = 125° 57′ 14″

And (Rule 2)

As sin. C	125° 57′ 14″	A. C. 0.091788
: sin. B	24° 15′ 17″	9.613624
:: AB	325	2.511883
: AC	164.93	2.217295

Rule 2.

As sin. C		A. C. 0.091788
: sin. A	29° 47′ 29″	9.696219
:: AB		2.511883
: BC	199.48	2.299890

Ex. 5.

As AC	106.49	A. C. 7.972691
: AB	127.54	2.105646
:: sin. B	52° 27′ 18″	9.899205
: sin. C	108° 16′ 3″	9.977542

As sin. B		A. C. 0.100795
: sin. A	19° 16′ 39″	9.518703
:: AC		2.027309
: BC	44.34	2.646807

Ex. 6. Rule 1.

As AC	398.47	A. C. 7.399605
: AB	527.63	2.722330
:: sin. B	43° 29′ 11″	9.837703
: sin. C	65° 40′ 44″	9.959638
or	114° 19′ 16″	

C *acute.*

Rule 2.

As sin. B	43° 29′ 11″	A. C. 0.162297
: sin. A	70° 50′ 5″	9.975237
:: AC		2.600395
: BC	546.93	2.737929

C *obtuse.*

As sin. B		A. C. 0.162297
: sin. A	22° 11′ 33″	9.577169
:: AC		2.600395
: BC	218.71	2.339861

Article 140.

Ex. 1.

As AB + BC	1875.10	A. C. 6.726976
: AB − BC	176.04	2.245611
:: tan. $\frac{C+A}{2}$	57° 8′ 20″	10.189790
: tan. $\frac{C-A}{2}$	8° 16′ 10″	9.162377

C = 65° 24′ 30″
A = 48° 52′ 10″

Rule 2, Art. 139.

As sin. A		A. C. 0.123083
: sin. B	65° 43′ 20″	9.959787
:: BC	849.53	2.929179
: AC	1028.13	3.012049

Ex. 3.

As AB + AC	373.39	A. C. 7.427838
: AB − AC	61.47	1.788663
:: tan. $\frac{C+B}{A}$	65° 20′ 30″	10.338123
: tan. $\frac{C-B}{A}$	19° 43′ 42″	9.554624

C = 85° 4′ 12″
B = 45° 36′ 48″

As sin. C	85° 4′ 12″	A. C. 0.001610
: sin. A	49° 19′	9.879855
:: AB	217.43	2.337320
: BC	165.495	2.218785

Article 141.

Ex. 2.

As AC		A. C. 6.270585
: BC		3.964217
:: rad.		10.000000
: tan. x	59° 47′ 5″	10.234802
	45°	
tan. $x - 45°$	14° 47′ 5″	9.421483
tan. $\frac{A+B}{2}$	58° 21′ 18″	10.210207
tan. $\frac{A-B}{2}$	23° 10′ 58″	9.631690
A	81° 32′ 16″	

As sin. A		A. C. 0.004754
: sin C.	63° 17′ 24″	9.950993
:: BC		3.964217
: AB	8317	3.919964

Ex. 3.

As AB		A. C. 8.036575
: BC		2.416347
:: rad.		10.000000
: tan. x	70° 35′ 9″	10.452922
	45°	
tan. $x - 45°$	25° 35′ 9″	9.680169
tan. $\frac{A+C}{2}$	25° 9′	9.671634
tan. $\frac{A-C}{2}$	12° 40′ 11″	9.351803
A	37° 49′ 11″	

As sin. A		A. C. 0.212413
: sin. B	129° 42′	9.886152
:: BC		2.416341
: AC	327.27	2.514912

Article 142.

Ex. 2. Here

As AB	214	A. C. 7.669586
: AC + BC	362	2.558709
:: AC − BC	28	1.447158
: AD − DB	47.364	1.675453

Whence AD = 130.682 and BD = 83.318.

Then (Art. 137, Rule 2,)

As AC	195	A. C. 7.709965
: AD	130.682	2.116216
:: rad.		10.000000
: cos. A	47° 55′ 13″	9.826181

And

As BC	167	A. C. 7.777284
: BD	83.318	1.920739
:: rad.		10.000000
: cos. B	60° 4′ 19″	9.698023

Ex. 3.

As AB	51.67	A. C. 8.286762
: AC + CB	71.11	1.851931
:: AC − CB	16.79	1.225051
: AD − DB	23.107	1.363744

Hence AD = 37.3885 and BD = 14.2815.

Then

As AC	43.95	A. C. 8.357041
: AD	37.3885	1.572738
:: rad.		10.000000
: cos. A	31° 42′ 42″	9.929779

And

As BC	27.16	A. C. 8.566070
: BD	14.2815	1.154774
:: rad.		10.000000
: cos. B	58° 16′ 34″	9.720844

Article 143.

Ex. 2. Here the half-sum of the sides is 288. Whence

As	AB	167	A. C. 7.777284
	AC	214	" " 7.669586
:	$\frac{1}{2}s$	288	2.459392
	$\frac{1}{2}s - BC$	93	1.968483
: :	rad.^{2}		20.000000
:	cos.$^2 \frac{1}{2}$ A		2)19.874745
	cos. $\frac{1}{2}$ A	30° 2′ 11″	9.937372
	A	60° 4′ 22″	

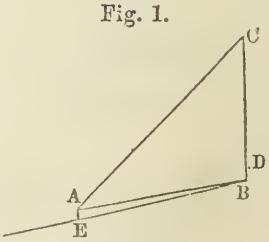
Ex. 3.

	BC 27.16		
	AB 51.67		A. C. 8.286762
	AC 43.95		8.357041
	2)122.78		
$\frac{1}{2}s$	61.39		1.788098
$\frac{1}{2}s - BC$	34.23		1.534407
			2)19.966308
cos. $\frac{1}{2}$ A		15° 51′ 20″	9.983154
A		31° 42′ 40″	

MISCELLANEOUS PROBLEMS.

Problem 6. — *Construction.* Draw BC (Fig. 1) to represent the tower. At B make CBA = 98° 19′. Make BD = 5 feet. With D as a centre and a radius of 75, describe an arc cutting BA in A. Draw AE and BE parallel to BD and AD; then will AE be the position of the instrument. Draw AC, making DAC = 41° 18′ 45″, and C will be the top of the tower.

Fig. 1.



Calculation.

As sin. C	40° 22′ 15″	A. C. 0.188605
: sin. BAC	41° 18′ 45″	9.819653
:: AB	75	1.875061
: BC	76.44	1.873319

In this solution, AB is supposed to be equal to BE. Without this supposition the calculation would be as follows:—

As EB (75) : EA (5) :: sin. EAB (98° 19′) : sin. ABE (3° 46′ 56″);

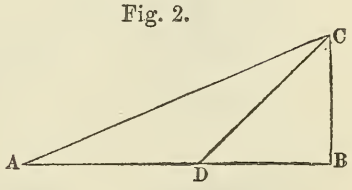
And As sin. EAB : sin. AEB (77° 54′ 4″) :: EB : AB (74.11);

Then As sin. C (40° 22′ 15″) : sin. BAC (41° 18′ 45″) :: AB : BC = 75.54.

Problem 7. Fig. 2.

Here ACD = BDC − BAC = 14° 27′ 15″. Then,

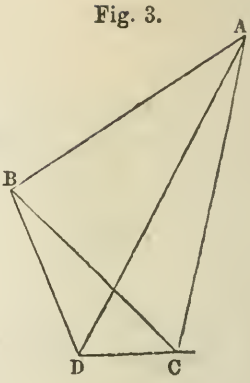
Fig. 2.



As sin. ACD	14° 27′ 15″	A. C. 0.602745
: sin. DAC	24° 18′	9.614385
:: AD	100	2.000000
: DC		2.217130
cos. CDB	38° 45′ 15″	9.892005
CB	128.57	2.109135
DC		2.217130
sin. CDB		9.796560
BC	103.2	2.013690
Height of inst.	4.75	
	107.95 = height required.	

Problem 8.

Fig. 3.



First. To find AD.

As sin. CAD	15° 51′ 45″	A. C.	0.563314
: sin. ACD	103° 47′		9.987310
:: CD	35.75		1.552276
: AD			2.103900

Second. To find BD.

As sin. CBD	24° 7′	A. C.	0.388706
: sin. BCD	45° 29′ 30″		9.853180
:: CD	35.75		1.553276
: BD			1.795162

Third. To find ABD and AB.

As BD		A. C.	8.204838
: AD			2.103900
:: rad.			10.000000
: tan. x	63° 50′ 22″		10.308738

And

As rad.		A. C.	0.000000
: tan. $x - 45°$	18° 50′ 22″		9.533005
:: tan. $\frac{ABD + BAD}{2}$	64° 58′ 52½″		10.330956
: tan. $\frac{ABD - BAD}{2}$	36° 10′ 10½″		9.863961
ABD	101° 9′ 3″		

As sin. ABD		A. C.	0.008277
: sin. ADB	50° 2′ 15″		9.884492
:: AD			2.103900
: AB	99.236		1.996669

Problem 9. Fig. 75. Surveying.

First. To find AD.

As sin. CAD	61° 18′	A. C. 0.056928
: sin. ACD	68° 15′	9.967927
:: CD	7.75	0.889302
: AD		0.914157

Second. To find BD and BAD.

As sin. DBE	61° 20′	A. C. 0.056790
: sin. DEB	75° 10′	9.985280
:: DE	7.92	0.898725
: BD		0.940795
AD		0.914157
tan. x	46° 45′ 22″	10.026638

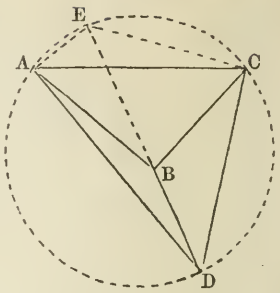
As rad.		A. C. 0.000000
: tan. $x - 45°$	1° 45′ 22″	8.486565
:: tan. $\frac{BAD + ABD}{2}$	33° 37′	9.822703
: tan. $\frac{BAD - ABD}{2}$	1° 10′ 4″	8.309268
BAD	34° 47′ 4″	

Lastly. To find AB.

As sin. BAD	34° 47′ 4″	A. C. 0.243752
: sin. BDA	112° 46′	9.964773
:: BD		0.940795
: AB	14.10 ch.	1.149320

Problem 10. The construction is as in Problem 4, page 112, Surveying.

Fig. 4.



First. To find BAC.

BC	4153		
AC	5916	A. C.	6.227972
AB	4596	" "	6.337620
	2)14665		
	7332.5		3.865252
	3179.5		3.502359
			2)19.933203
cos. $\frac{1}{2}$ BAC	22° 11′ 1″		9.966601
BAC	44° 22′ 2″		

Second. To find AE and ABE.

As sin. AEC	136° 26′ 30″	A. C.	0.161722
: sin. ACE	19° 14′ 30″		9.517926
:: AC			3.772028
: AE			3.451676
AB	4596		3.662380
tan. x	58° 23′ 1″		10.210704

As rad.		A. C.	0.0000000
: tan. $x - 45°$	13° 23′ 1″		9.376451
:: tan. $\frac{AEB + ABE}{2}$	55° 39′ 29″		19.165435
: tan. $\frac{AEB - ABE}{2}$	19° 12′ 2″		9.541886
ABE	36° 27′ 27″		

BAD = ABE − ADB = 17° 12′ 57″.

Third. To find AD and BD.

As sin. ADB	19° 14′ 30″	A. C.	0.482074
: sin. ABD	143° 32′ 33″		9.773952
:: AB			3.662380
: AD	8287.2		3.918406

And

As sin. ABD		A. C. 0.482074
: sin. BAD	17° 12′ 57″	9.471251
:: AB		3.662380
: BD	4127.7	3.615705

Lastly. To find DC.

As sin. ADC	43° 33′ 30″	A. C. 0.161723
: sin. CAD	61° 34′ 59″	9.944240
:: AC	5916	3.772028
: CD	7550.8	3.877991

Problem 11.

First. To find CA and AG.

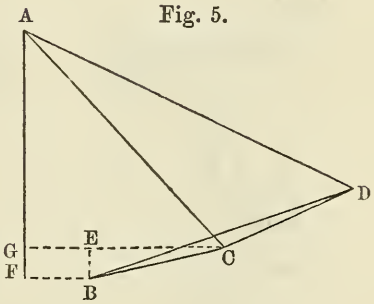


Fig. 5.

As sin. CAD	21° 48′ 45″	A. C. 0.429959
: sin. ADC	49° 48′ 30″	9.883030
:: CD	800	2.903090
: CA		3.216079
sin. ACG	47° 22′ 15″	9.866732
AG	1210.07	3.082811

Second. To find BC and BE.

As sin. CBD	5° 45′	A. C. 0.999184
: sin. BDC	6°	9.019235
:: CD	800	2.903090
: CB		2.921509
sin. ECB	12° 30′	9.335337
EB	180.65	2.256846
AG	1210.07	
AF	1390.72 = height required.	

Problem 12. Fig. 76. Surveying.—The measured angles being the horizontal and vertical angles, we have the following data,—ABEF being a horizontal plane:—AB = 252.28 feet, ABE = 80° 51′ 30″, BAE = 82° 54′ 30″, ABF = 74° 37′, BAF = 89° 24′, and the elevations as in the problem.

First. To find AE, AF, AFE, and EF.

As sin. AEB	16° 14′	A. C. 0.553541
: sin. ABE	80° 51′ 30″	9.994448
:: AB	252.28	2.401883
: AE		2.949872

And

As sin. AFB	15° 59′	A. C. 0.560103
: sin. ABF	74° 37′	9.984155
:: AB		2.401883
: AF		2.946141
AE		2.949872
tan. x	45° 14′ 46″	10.003731
tan. $x - 45°$	0° 14′ 46″	7.633005
tan. $\frac{AFE + AEF}{2}$	86° 45′ 15″	11.246335
tan. $\frac{AFE - AEF}{2}$	4° 19′ 53″	8.879340

Then

As sin. AFE	91° 5′ 8″	A. C. 0.000078
: sin. EAF	6° 29′ 30″	9.053304
:: AE		2.949872
: EF	100.75	2.003254

Second. To find EC, EH, and CH.

As rad.		A. C. 0.000000
: tan. EAC	3° 45′	8.816529
:: EA		2.949872
: EC	58.40	1.766401

And

As rad.		A. C. 0.000000
: tan. EAH	9° 25′	9.219710
:: EA		2.949872
: EH	147.77	2.169582
EC	58.40	
CH	89.37	

Third. To find FD, FG, and DG.

As rad.		A. C. 0.000000
: tan. DAF	3° 54′	8.833613
:: FA		2.946141
: FD	60.22	1.779754

And

As rad.		A. C. 0.000000
: tan. GAF	10° 29′ 30″	9.267614
:: FA		2.946141
: FG	163.59	2.213755
FD	60.22	
DG	103.37	

CD is sensibly equal to EF = 100.75.

Problem 13. Fig. 77. Surveying.

First. To find AE and EB.

As sin. AEB	42° 40′	A. C. 0.168942
: sin. ABE	72° 43′	9.979934
:: AB	7.37	0.867467
: AE	10.38	1.016343

And

As sin. AEB		A. C. 0.168942
: sin. BAE	64° 37′	9.955909
:: AB		0.867467
: BE	9.8247	0.992318

Second. To find CE.

As BE + BC	17.7747	A. C. 8.726427
: BE − BC	.8747	−1.941859
:: tan. $\frac{BCE + BEC}{2}$	15° 14′	9.435078
: tan. $\frac{BCE - BEC}{2}$	43′ 37″	8.103364
BCE	15° 57′ 37″	
BEC	14° 30′ 23″	

And

As sin. BEC	14° 30′ 23″	A. C. 0.601213
: sin. CBE	149° 32′	9.705040
:: BC	8.95	0.951823
: CE		1.258076

Third. To find DF and CF.

As sin. DFC	27° 47′	A. C. 0.331494
: sin. DCF	69° 38′	9.971964
:: DC	9.33	0.969882
: DF	18.765	1.273340

And

As sin. DFC		A. C. 0.331494
: sin. CDF	82° 35′	9.996351
:: DC		0.969882
: CF		1.297727

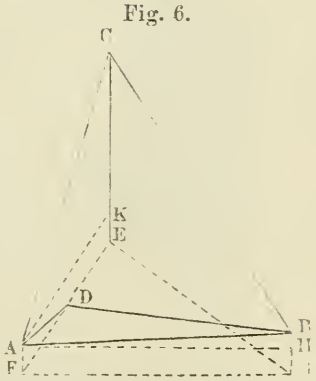
Fourth. To find EF.

CF		1.297727
CE		1.258076
tan. x	47° 36′ 43″	10.039651
	45°	
tan. $x - 45°$	2° 36′ 43″	8.659140
tan. $\frac{CEF + CFE}{2}$	28° 5′ 19″	9.727293
tan. $\frac{CEF - CFE}{2}$	1° 23′ 41″	8.386433
CEF	29° 29′ 0″	
CFE	26° 41′ 38″	

As sin. CEF	29° 29′	A. C. 0.307885
: sin. ECF	123° 49′ 23″	9.919476
:: CF		1.297727
: EF	33.50	1.525088

AEF = AEB + BEC + CEF = 86° 39′ 23″.
EFD = CFD + CFE = 54° 28′ 38″.

Fig. 6.



Problem 14.

First. To find AH and BH.

As rad.		A. C. 0.000000
: cos. BAH	3° 14′ 30″	9.999304
:: AB	850	2.929419
: AH = FG		2.928723

And

As rad.		A. C. 0.000000
: sin. BAH		8.752412
:: AB		2.929419
: BH	48.07	1.681831

Second. To find GE and FE.

As sin. GEF	21° 27′ 30″	A. C. 0.436727
: sin. EFG	87° 49′	9.999685
:: FG		2.928723
: GE	2318.1	3.365135

And

As sin. GEF		A. C. 0.436727
: sin. FGE	70° 43′ 30″	9.974947
:: FG		2.928723
: FE	2189.8	3.340397

Third. To find FD, GD, AF, and BG.

As sin. FDG	44° 32′	A. C. 0.154081
: sin. FGD	47° 39′	9.868670
:: FG		2.928723
: FD	894.3	2.951474

And

As sin. FDG		A. C. 0.154081
: sin. DFG	87° 49′	9.999685
:: FG		2.928723
: DG	1209.2	3.082489

DF		2.951474
tan. ADF	3° 25′ 45″	8.777584
AF	53.59	1.729058
BH	48.07	
BG	101.66	

Fourth. To find CK and CE.

AK		3.340397
tan. CAK	35° 27′	9.852466
CK	1559.1	3.192863
AF	53.59	
CE	1612.69	

CHAPTER IV.

CHAIN SURVEYING.

Article 251.

Ex. 2.

$$\frac{17.25 + 16.43 + 14.65}{2} = \frac{48.33}{2} = 24.165 = \text{half sum of sides.}$$

$\frac{1}{2}$ sum		24.165	1.383187
$\frac{1}{2}$ sum	−17.25	6.915	0.839792
$\frac{1}{2}$ sum	−16.43	7.735	0.888460
$\frac{1}{2}$ sum	−14.65	9.515	0.978409
			2) 4.089848
Area, 110.9 ch. = 11 A., 0 R., 14.4 P.			2.044924

Ex. 3. $\frac{19.58 + 16.92 + 12.76}{2} = \frac{49.26}{2} = 24.63 = \frac{1}{2}\,s.$

$\frac{1}{2}\,s$		24.63	1.391464
$\frac{1}{2}\,s$	−19.58	5.05	0.703291
$\frac{1}{2}\,s$	−16.92	7.71	0.887054
$\frac{1}{4}\,s$	−12.76	11.87	1.074451
			2) 4.056260
106.69 ch. = 10 A., 2 R., 27.04 P.			2.028130

Article 254.

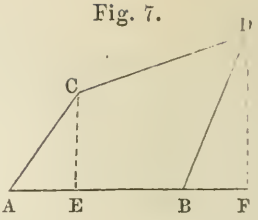
Ex. 2. Fig. 107. Surveying.

2 AED	= AE . ED	= 4.15 × 8.25	= 34.2375
2 EFCD	= EF . (ED + CF)	= 5.02 × 17.61	= 88.4022
2 CFB	= FB . FC	= 2.26 × 9.36	= 21.1536
			2) 143.7933
			71.89665 ch.

= 7 A., 0 R., 30.3 P.

Ex. 3.

Fig. 7.

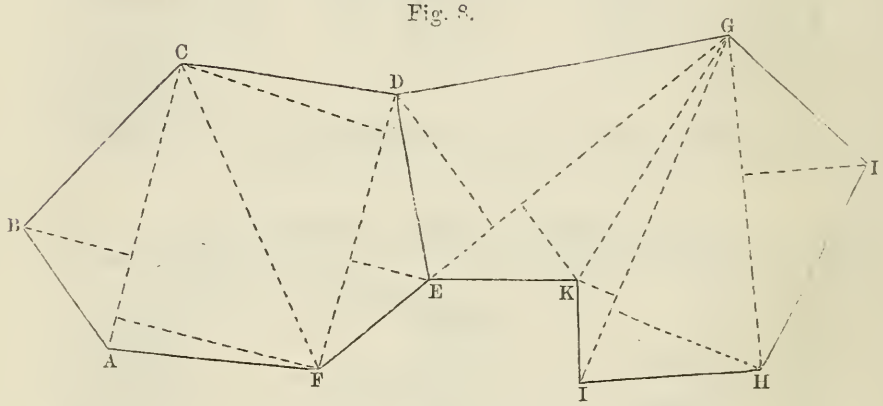


$2\,AEC = AE.EC = 3.76 \times 5.68 = 21.3568$
$2\,CEFD = EF.(CE + FD) = 9.89 \times 14.60 = 144.3940$
165.7508
$2\,BDF = BF.FD = 3.98 \times 8.92 = 35.5016$
$2)\,130.2492$
65.1246 ch.

$= 6$ A., 2 R., 1.99 P.

Article 255.

Fig. 8.



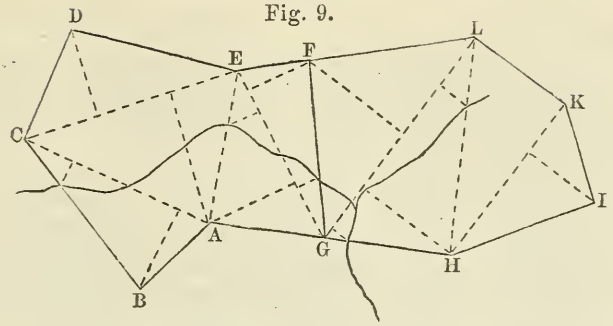
	Bases.	Perpendiculars.	Areas.
First field. 2 ABCF	9.90	11.10	109.8900
2 FCDE	9.70	10.56	102.4320
			2) 212.3220

10 A., 2 R., 18.576 P. = 106.161 ch.

Second field. 2 EKGD	13.15	8.75	115.0625
2 KGHI	12.85	7.20	92.5200
2 GHL	11.50	4.32	49.6800
			2) 257.2625

12 A., 3 R., 18.1 P. = 128.63125 ch.

Fig. 9.



First field.

Areas.	Bases.	Perpendiculars.	Areas.
2 ABC	11.50	4.82	55.4300
2 ACDE	12.55	12.65	158.7575
2 AEFG	10.51	9.20	96.6920

2) 310.8795

15 A., 2 R., 7 P. = 155.43975 ch.

Second field.

2 FGHL	13.95	13.00	181.3500
2 HIKL	10.80	10.85	117.1800

2) 298.5300

14 A., 3 R., 28.24 P. = 149.265 ch.

Ex. 2. First tract.

Article 256.

	Distances.	Offsets.	Inter. Dist.	Sum of Offsets.	Double Areas.
	0.00	0.00			
Left hand areas.	7.90	6.00	7.90	6.00	47.4000
	19.20	4.00	11.30	10.00	113.0000
	21.45	3.75	2.25	7.75	17.4375
	31.27	0.00	9.82	3.75	36.8250
	0.00	0.00			
	2.00	4.65	2.00	4.65	9.3000
Right hand areas.	8.40	5.16	6.40	9.81	62.7840
	14.85	5.23	6.45	10.39	67.0155
	20.70	6.42	5.85	11.65	68.1525
	25.90	4.76	5.20	11.18	58.1360
	31.27	0.00	5.37	4.76	25.5612

2) 505.6117

25 A., 1 R., 4.89 P. = 252.80585 ch.

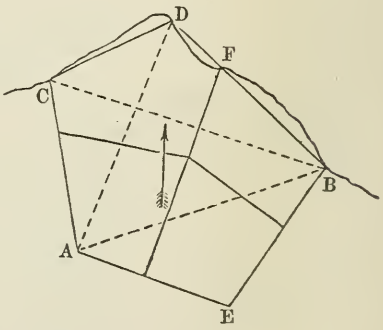
Second tract.

Distances.	Offsets.	Inter. Dist.	Sum of Offsets.	Double Areas.
0.00	0.00			
1.75	2.37	1.75	2.37	4.1475
10.26	7.29	8.51	9.66	82.2066
21.37	4.35	11.11	11.64	129.3204
28.46	5.94	7.09	10.29	72.9561
40.25	0.00	11.79	5.94	70.0326
0.00	0.00			
4.29	6.23	4.29	6.23	26.7267
15.48	3.19	11.19	9.42	105.4098
32.54	8.26	17.06	11.45	195.3370
36.17	7.92	3.63	16.18	58.7334
40.25	0.00	4.08	7.92	32.3136
				2) 777.1837

38 A., 3 R., 17.47 P. = 388.59185 ch.

Article 257.

Fig. 10.



Ex. 2.

First. To find ABC.

$$\frac{21.69 + 23.72 + 14.50}{2} = \frac{59.91}{2} = 29.955.$$

$\frac{1}{2}\,s$	29.955	1.476469
$\frac{1}{2}\,s$ — AB	8.265	0.917243
$\frac{1}{2}\,s$ — BC	6.235	0.794836
$\frac{1}{2}\,s$ — AC	15.455	1.189068
		2) 4.377616
ABC	154.457	2.188808

Second. To find ABE.

$$\frac{21.69 + 13.96 + 13.44}{2} = \frac{49.09}{2} = 24.545.$$

$\frac{1}{2}\,s$	24.545	1.389963
$\frac{1}{2}\,s - AB$	2.855	0.455606
$\frac{1}{2}\,s - BE$	10.585	1.024691
$\frac{1}{2}\,s - AE$	11.105	1.045518
		2) 3.915778
ABE	90.759	1.957889

To find CBD.

$$\frac{23.72 + 17.40 + 11.32}{2} = \frac{52.44}{2} = 26.22.$$

$\frac{1}{2}\,s$	26.22	1.418633
$\frac{1}{2}\,s - CB$	2.50	0.397940
$\frac{1}{2}\,s - BD$	8.82	0.945469
$\frac{1}{2}\,s - CD$	14.90	1.173186
		2) 3.935228
CBD	92.814	1.967614

Offsets.

Base.	Distance.	Offsets.	Int. Dist.	Sum of Offsets.	Double Areas.
	000				
	4.80	.75	4.80	.75	3.6000
CD	7.39	.97	2.59	1.72	4.4548
	8.96	.72	1.57	1.69	2.6533
	10.54	.55	1.58	1.27	2.0066
	11.32		.78	.55	.4290
	5.51	.00			
	6.52	.50	1.01	.50	.5050
	8.24	.45	1.72	.95	1.6340
FB	10.16	.87	1.92	1.32	2.5344
	12.37	.35	2.21	1.22	2.6962
	14.14	.63	1.77	.98	1.7346
	17.40		3.26	.63	2.0538
					24.3017
	00				
	2.95	.75	2.95	.75	2.2125
DF	4.52	.75	1.57	1.50	2.3550
	5.51		.99	.75	.7425

Subtractive 5.3100

2) 18.9917

9.4958

ABC 154.457

ABE 90.759

CBD 92.814

Area, 34 A., 3 R., = 347.5258 ch.

CHAPTER V.

COMPASS SURVEYING.

Article 326.

Ex. 1. Fig. 127. Surveying.

To find BD, AD, BAD, and ABD.

As sin. DBE	37° 55′	A. C. 0.211468
: sin. DEB	120° 45′	9.934199
: : DE	18	1.255273
: DB		1.400940

As sin. DAE	46° 35′	A. C. 0.138839
: sin. AED	61° 20′	9.943210
: : DE		1.255273
: AD		1.337322
BD		1.400940
tan. x	49° 10′ 54″	10.063618
tan. $x - 45°$	4° 10′ 54″	8.863999
tan. $\frac{\text{BAD} + \text{ABD}}{2}$	64° 37′ 30″	10.323947
tan. $\frac{\text{BAD} - \text{ABD}}{2}$	8° 45′ 47″	9.187946
BAD	73° 23′ 17″	
ABD	55° 51′ 43″ = DGC.	

DGF = 180° − 55° 51′ 43″ = 124° 8′ 17″.

To find DC, DG, and CG.

As sin. DCE	46° 35′	A. C. 0.138839
: sin. DEC	26° 50′	9.654558
: : DE		1.255273
: DC		1.048670

As sin. DGC	55° 51′ 43″	A. C. 0.082134
: sin. DCG	38° 53′ 17″	9.797821
: : DC		1.048670
: DG	8.48	0.928625

As sin. DGC		A. C. 0.082134
: sin. CDG	85° 15′	9.998506
: : DC		1.048670
: CG	13.47	1.129310

Ex. 2. Fig. 120. Surveying.

To find BC, CA, CAB, and ABC.

As sin. CBD	42° 28′	A. C. 0.170593
: sin. CDB	47° 29′	9.867515
: : DC	7.00	0.845098
: CB		0.883206

As sin. CAD	47° 18′	A. C. 0.133763
: sin. ADC	100° 25′	9.992783
: : CD		0.845098
: AC		0.971644
CB		0.883206
tan. x	50° 47′ 38″	10.088438
tan. $x - 45°$	5° 47′ 38″	9.006330
tan. $\frac{ABC + BAC}{2}$	61° 7′	10.258336
tan. $\frac{ABC - BAC}{2}$	10° 25′ 20″	9.264666
ABC	71° 32′ 20″	
BAC	50° 41′ 40″	

$$ACF = BFC - BAC = 16° 53′ 20″.$$

$$DCF = DCA + ACF = 49° 10′ 20″.$$

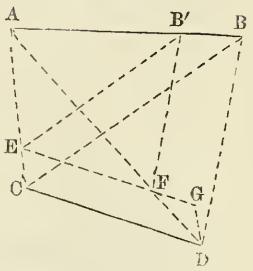
To find AF and FC.

As sin. AFC	67° 35′	A. C. 0.034124
: sin. ACF	16° 53′ 20″	9.463171
: : AC		0.971644
: AF	2.94	0.468939

And as sin. AFC		A. C. 0.034124
: sin. CAF	50° 41′ 40″	9.888617
: : AC		0.971644
: CF	7.84	0.894385

Ex. 3. *Construction.*—Take AB′ any length. At A and B′ make the angles B′AE, B′AF, AB′E, and AB′F equal to the given angles. Join EF: produce it to G, making EG = 10.78. Through G draw GD parallel to AE, meeting AF in D. Draw DB parallel to FB′, and BC parallel to B′E. Then will AB be the line and CD the known base.

Fig. 11.



By Art. 316 we have

AD : AC : : sin. ABD . sin. ACB : sin. ADB . sin. ABC;

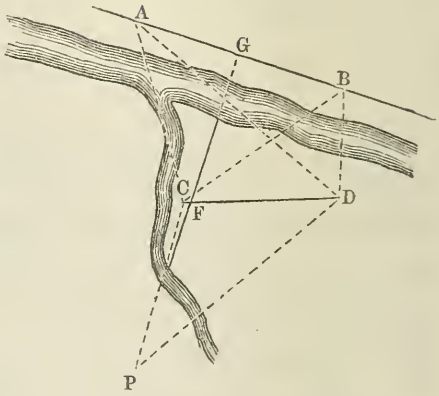
whence

As AD or { sin. ABD	80° 47′	A. C. 0.005643
{ sin. ACB	60° 54′	" " 0.058602
: AC or { sin. ADB	52° 43′	9.900722
{ sin. ABC	37° 23′	9.783292
: : rad.		10.000000
: tan. x	29° 15′ 10″	9.748259
tan. $45° - x$	15° 44′ 50″	9.450213
tan. $\frac{ACD + ADC}{2}$	72° 23′ 30″	10.498422
tan. $\frac{ACD - ADC}{2}$	41° 37′ 11″	9.948635
ACD	114° 0′ 41″	

And (Art. 316)

As	sin. CAD	35° 13′	A. C. 0.239073
	sin. ABD	80° 47′	" " 0.005643
:	sin. ACD	114° 0′ 41″	9.960691
	sin. ADB	52° 43′	9.900722
: :	CD	10.78	1.032619
:	AB	13.76	1.138748

Fig. 12.



Ex. 4. To find BD, AD, and ABD.

As sin. CBD	54° 25′	A. C. 0.089765
: sin. BCD	33° 18′	9.739590
: : CD	15	1.176091
: BD		1.005446

As sin. CAD	36° 2′	A. C. 0.230434
: sin. ACD	103° 10′	9.988430
: : CD		1.176091
: AD		1.394955
BD		1.005446
tan. x	67° 48′ 45″	10.389509
tan. $x - 45°$	22° 48′ 45″	9.623888
tan. $\frac{ABD + BAD}{2}$	64° 15′ 30″	10.316805
tan. $\frac{ABD - BAD}{2}$	41° 6′	9.940693
ABD	105° 21′ 30″	

GFD = 360° − (FDB + DBG + BGF) = 72° 21′ 30″.

FPD = GFD − PDF = 33° 31′ 30″.

To find PD and FD.

As sin. CPD	35° 40′	A. C. 0.234280
: sin. PCD	105° 30′	9.983911
: : CD	15	1.176091
: PD		1.394282
And as sin. PFD	72° 21′ 30″	A. C. 0.020921
: sin. FPD	33° 31′ 30″	9.742175
: : PD		1.394282
: FD	14.37	1.157378

Ex. 5. Fig. 118. Surveying.

To find CA, CB, CAB, and CBA.

As sin. CAD	41° 15′	A. C. 0.180887
: sin. CDA	43° 45′	9.839800
: : CD	10	1.000000
: CA		1.020687
As sin. CBD	55° 1′	A. C. 0.086547
: sin. CDB	87° 39′	9.999635
: : CD		1.000000
: CB		1.086182
CA		1.020687
tan. x	49° 18′ 15″	10.065495
tan. $x - 45°$	4° 18′ 15″	8.876584
tan. $\frac{CAB + ABC}{2}$	61° 10′	10.259233
tan. $\frac{CAB - ABC}{2}$	7° 47′ 6″	9.135817
CAB	68° 57′ 6″	
ABC	53° 22′ 54″	

To find CE, and AE, and DCE.

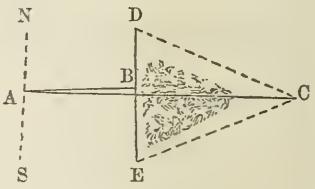
As sin. AEC	81° 43′	A. C. 0.004554
: sin. CAE	29° 19′ 54″	9.690076
: : AC		1.020687
: CE	5.192	0.715317

And as sin. AEC		A. C. 0.004554
: sin. ACE	68° 57′ 6″	9.970011
: : AC		1.020687
: AE	9.89	0.995252

DCE = DCA + ACE = 95° + 68° 57′ 6″ = 163° 57′ 6″.

Ex. 6. To find DC.

Fig. 13.



As sin. DCE	45° 56′	A. C. 0.143554
: sin. DEC	67° 25′	9.965353
: : DE	9.25	0.966142
: DC	11.89	1.075049

To find BC.

As DC + DB	16.14	A. C. 8.792096
: DC − DB	7.64	0.883093
: : tan. $\frac{DBC + DCB}{2}$	56° 40′ 30″	10.182102
: tan. $\frac{DBC - DCB}{2}$	35° 45′ 5″	9.857291
DBC	92° 25′ 35″	

And as sin. DCB	20° 55′ 25″	A. C. 0.447182
: sin. BDC	66° 39′	9.962890
: : DB	4.25	0.628389
: BC	10.93	1.038461

Deflection of BC = 92° 25′ 35″ − 90° = 2° 25′ 35″.

To find BAC and AC.

As BC + BA	18.43	A. C. 8.734475
: BC − BA	3.43	0.535294
: : tan. $\frac{BAC + BCA}{2}$	1° 12′ 47″	8.325822
: tan. $\frac{BAC - BCA}{2}$	13′ 33″	7.595591
BAC	1° 26′ 20″	
Bearing of AB	N. 87° E.	
" AC	N. 88° 26′ E.	

And

As sin. BAC	1° 26′ 20″	A. C. 1.600141
: sin. ABC	2° 25′ 35″	8.626705
: : BC		1.038461
: AC	18.42	1.265307

Article 333.

Ex. 1.

As rad.		A. C. 0.000000
: sin. bearing	56° 15′	9.919846
: : Dist.	37.56	1.574726
: Dep.	31.23	1.494572

As rad.		A. C. 0.000000
: cos. bearing		9.744739
: : Dist.		1.574726
: Lat.	20.87	1.319465

Ex. 2.

As Dist.	52.95	A. C. 8.276134
: Lat.	36.17	1.558349
: : rad.		10.000000
: cos. bearing	46° 55′	9.834483

As rad.		A. C. 0.000000
: sin. bearing		9.863538
: : Dist.		1.723866
: Dep.	38.67	1.587404

Ex. 3.

As Lat.	19.25	A. C. 8.715569
: Dep.	26.45	1.422426
: : rad.		10.000000
: tan. bear.	53° 57′	10.137995

As cos. bear.		A. C. 0.230260
: rad.		10.000000
: : Lat.		1.284431
: Dist.	32.71	1.514691

Ex. 4.

As sin. bear.	33° 30′	A. C. 0.258111
: rad.		10.000000
: : Dep.	18.33	1.263162
: Dist.	33.21	1.521273

As rad.		A. C. 0.000000
: cos. bear.		9.921107
: : Dist.		1.521273
: Lat.	27.69	1.442380

Article 334.

Ex. 4.

Dist.	Lat.	Dep.
20	8.924	17.899
7	3.123	6.264
.4	.178	.358
.09	40	80
	12.265	24.601

Ex. 5.

Dist.	Lat.	Dep.
20	11.256	16.532
7	3.940	5.786
	15.196	22.318

Ex. 6.

Dist.	Lat.	Dep.
100	9.150	99.580
20	1.830	19.916
3	.274	2.987
.5	46	498
.06	5	60
	11.305	123.041

Ex. 7.

Dist.	Lat.	Dep.
90	81.733	37.679
7	6.357	2.931
.5	.454	.209
.06	54	25
	88.598	40.844

A slight inspection of the rejected figures in the columns for latitude in the fourth and sixth examples will show that the second decimal should be increased by one in each case.

Article 335.

Ex. 1.

Bear.	Dist.	Lat.	Dep.	Bear.	Lat.	Dep.
$76\frac{1}{2}°$	30	7.003	29.171	$76\frac{3}{4}°$	6.876	29.201
	9	2.101	8.751		2.063	8.760
		9.104	37.922		8.939	37.961
		−.132	+.033		9.104	37.922
76° 42′	39	8.972	37.955	diff.	.165	.039
	.7	.161	.681	$\frac{12}{15}$ of diff.	.132	.033
	.06	14	58			
		9.147	38.694			

Ex. 2.

Bear.	Dist.	Lat.	Dep.	Bear.	Lat.	Dist.
37°	60	47.918	36.109	$37\frac{1}{4}°$	47.760	36.318
	3	2.396	1.805		2.388	1.816
		50.314	37.914		50.148	38.134
		− .100	+ .132		50.314	37.914
37° 9′	63	50.214	38.046	diff.	.166	.220
	.4	.319	.241	$\frac{9}{15}$ diff.	.100	.132
	.05	40	30			
		50.573	38.317			

Ex. 3.

Bear.	Dist.	Lat.	Dep.	Bear.	Lat.	Dep.
29° 15′	100	87.250	48.862	29° 30′	87.036	49.242
	20	17.450	9.772		17.407	9.848
	3	2.617	1.466		2.611	1.477
		107.317	60.100		107.054	60.567
		−.035	+.062		107.317	60.100
29° 17′	123	107.282	60.162	diff.	.263	.467
	.7	.611	.343	$\frac{2}{15}$ diff.	.035	.062
	.05	44	24			
		107.937	60.529			

Article 336.

Ex. 3.

Bearing 56° 7′	cosine .55750	sine .83017
60	33.4500	49.8102
3	1.6725	2.4905
.4	.2230	.3321
.08	466	664
	35.3921	52.6992

Ex. 4.

Bear. 52° 49′	cos. .60437	sin. .79671
100	60.437	79.671
30	18.131	23.901
6	3.626	4.780
.7	.423	.558
.05	.030	40
	82.647	108.950

Ex. 5.

Bear. 23° 47′	cos. .91508	sin. .40328
10	9.151	4.033
3	2.745	1.210
.6	.549	.242
.02	.018	8
	12.463	5.493

Article 338.

Ex. 2.

Bearings.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. E.	N.	S.	E.	W.
S. 49° W.	12.93		8.48		9.76	0	0		8.48		9.76
S. 88° W.	13.68		.47		13.67	1	0		.48		13.67
N. 25¼ W.	14.09	12.74			6.01	1	0	12.73			6.01
N. 43¼ E.	14.70	10.71		10.07		1	0	10.70		10.07	
N. 12½ W.	17.95	17.52			3.89	1	1	17.51			3.88
N. 88¾ E.	17.68	.39		17.68		1	1	.38		17.69	
S. 36½ E.	35.80		28.78	21.29		1	1		28.79	21.30	
S. 77¼ W.	16.15		3.56		15.75	1	1		3.57		15.74
	142.98	41.36	41.29	49.04	49.08						

Article 345.

Ex. 3.	Ex. 4.	Ex. 5.
AB N. 39° W.	S. 63° 29′ E.	S. 34° 12′ E.
75° 26′	29° 17′	75° 32′
114° 26′	S. 34° 12′ E.	S. 41° 20′ W.
180°		
BC S. 65° 34′ W.		

Article 346.

Ex. 5.	Ex. 6.	Ex. 7.
N. 46° 30′ E.	S. 63° W.	N. 82½° W.
S. 63° 29′ W.	N. 29° W.	N. 29¼° W.
16° 59′	92°	53¼° R.
180°	180°	
163° 1′ L.	88° R.	

Article 347.

Ex. 1.	Ex. 2.	Ex. 3.
BA S. 87° W.	BA N. 63° W.	DC S. 15° E.
BC S. 25° W.	BC N. 56° E.	DE N. 56° W.
ABC 62°	ABC 119°	41°
		180°
		CDE 139°

Article 348.

Ex. 2.

1st. N. 43° 25′ W.
N. 29° 48′ E.
N. 73° 13′ W.

2d. North.

3d. S. 80° E.
N. 29° 48′ E.
109° 48′
180°
N. 70° 12′ E.

4th. N. 89° 55′ E.
N. 29° 48′ E.
N. 60° 7′ E.

5th. S. 10° 13′ E.
N. 29° 48′ E.
S. 40° 1′ E.

6th. N. 63° 55′ W.
N. 29° 48′ E.
93° 43′ E.
180°
S. 86° 17′ W.

7th. S. 63° 45′ W.
N. 29° 48′ E.
S. 33° 57′ W.

8th. N. 57° 35′ W.
N. 29° 48′ E.
N. 87° 23′ W.

Ex. 3.

1st. S. 63° E.
N. $84\frac{1}{2}°$ W.
S. $21\frac{1}{2}°$ W.

2d. S. 47° E.
N. $84\frac{1}{2}°$ W.
S. $37\frac{1}{2}°$ W.

3d. S. $59\frac{1}{4}°$ W.
N. $84\frac{1}{2}°$ W.
$143\frac{3}{4}°$
180°
N. $36\frac{1}{4}°$ W.

5th. N. 12° W.
N. $84\frac{1}{2}°$ W.
N. $72\frac{1}{2}°$ E.

6th. N. $17\frac{1}{2}°$ E.
N. $84\frac{1}{2}°$ W.
102°
180°
S. 78° E.

7th. S. $29\frac{3}{4}°$ W.
N. $84\frac{1}{2}°$ W.
$114\frac{1}{4}°$
180°
N. $65\frac{3}{4}°$ W.

Article 350.

Ex. 3.

Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	S. $29\frac{3}{4}°$ E.	3.19		2.77	1.58	
2	S. $37\frac{1}{4}°$ W.	5.86		4.66		3.55
3	S. $39\frac{1}{4}°$ E.	11.29		8.74	7.14	
4	N. 53° E.	19.32	11.63		15.43	
5			(13.92)			(8.39)
6	S. $60\frac{3}{4}°$ W.	7.12		3.48		6.21
7	S. $29\frac{1}{4}°$ E.	2.18		1.90	1.07	
8	S. $60\frac{1}{2}°$ W.	8.12		4.00		7.07
			25.55	25.55	25.22	25.22

As Lat.	13.92	A. C. 8.856361
: Dep.	8.39	0.923762
: : rad.		10.000000
: tan. bearing	N. 31° 5′ W.	9.780123
As cos. bearing		A. C. 0.067315
: rad.		10.000000
: : Lat.		1.143639
: Dist.	16.26	1.210954

Ex. 4.

Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	N. 46° 40′ W.	18.41	12.63			13.39
2	N. 54° 30′ E	13.45	7.81		10.95	
3			(12.61)			(1.20)
4	S. 74° 55′ E.	17.58		4.57	16.97	
5	S. 47° 50′ E.	15.86		10.65	11.76	
6	S. 47° 25′ W.	16.36		11.07		12.05
7	S. 62° 35′ W.	14.69		6.76		13.04
			33.05	33.05	39.68	39.68

As Lat.	12.61	A. C. 8.899285
: Dep.	1.20	0.079181
:: rad.		10.000000
: tan. bearing	5° 26′	8.978466
As cos. bearing		A. C. 0.001956
: rad.		10.000000
:: Lat.		1.100715
: Dist.	12.67	1.102671

Ex. 5.

Sta.	Bearings.	Dist.	N.	S.	E.	W.
AC	S. 49¼° W.	9.30		6.07		7.05
CD	S. 32½° E.	10.25		8.64	5.51	
DE	S. 5¼° W.	6.75		6.72		.62
EB	N. 79¾° E.	8.10	1.44		7.97	
BA			(19.99)			(5.81)
			21.43	21.43	13.48	13.48

As Lat.	19.99	A. C. 8.699187
: Dep.	5.81	0.764176
:: rad.		10.000000
: tan. bearing	16° 12′	9.463363
As cos. bearing		A. C. 0.017596
: rad.		10.000000
:: Lat.		1.300813
: Dist.	20.82	1.318409

Ex. 6.

Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	S. 27½° E.	12.65		11.22	5.84	
2	S. 10¼° E.	23.45		23.08	4.17	
3	S. 14° W.	124.33		120.64		30.08
4	S. 67° E.	82.43		32.21	75.88	
5	S. 17° E.	96.35		92.14	28.17	
			(279.29)			(83.98)
				279.29	114.06	114.06

As Lat. 279.29 A. C. 7.553945
: Dep. 83.98 1.924176
:: rad. 10.000000
: tan. bearing S. 16° 44′ E. 9.478121

As cos. bearing A. C. 0.018791
: rad. 10.000000
:: Lat. 2.446055
: Dist. 291.64 2.464856

Article 351.

Ex. 2.

Sta.	Bearings.	Ch. Bear.	Dist.	N.	S.	E.	W.
1	N. 47° W.	N. 40° W.	16.55	12.68			10.64
2	N. 19° 5′ W.	N. 12° 5′ W.	11.48	11.23			2.40
3	N. W.		15.53	(14.45)			(5.69)
4	N. 23° E.	N. 30° E.	9.72	8.42		4.86	
5	N. 75¼° E.	N. 82¼° E.	14.00	1.89		13.87	
6	S. 7° E.	North.			(48.67)		
						18.73	18.73

As Dist.	15.53	A. C. 8.808829
: Dep.	5.69	0.755112
: : rad.		10.000000
: sin. ch. bear.	N. 21° 30′ W. S. 7° E.	9.563941
Bearing,	N. 28° 30′ W.	

As rad.		A. C. 0.000000
: cos. ch. bear.		9.968678
: : Dist.		1.191171
: Dep.	14.45	1.159849

Article 352.

Ex. 2.

Sta.	Bearings.	Ch. Bear.	Dist.	N.	S.	E.	W.
1	S. 29¾° E.	S. 9½° W.	3.19		3.15		.53
2	S. 37¼° W.	S. 76½° W.	5.86		1.37		5.70
3	S. 39¼° E.	South.	(11.28)		(11.28)		
4	N. 53° E.	S. 87¾° E.	19.33		.76	19.31	
5	N. 31° 5′ W.	N. 8° 10′ E.	(16.26)	(16.10)		(2.31)	
6	S. 60¾° W.	N. 80° W.	7.12	1.24			7.01
7	S. 29¼° E.	S. 10° W.	2.18		2.15		.38
8	S. 60½° W.	N. 80¼° W.	8.12	1.37			8.00
				18.71	18.71	21.62	21.62

As sin. ch. bear.	8° 10′	A. C. 0.847549
: rad.		10.000000
: : Dep.	2.31	0.363612
: Dist.	16.26	1.211161

As rad.		A. C. 0.000000
: cos. ch. bear.		9.995573
: : Dist.		1.211161
: Lat.	16.10	1.206734

Article 353.

Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	S. 29¾° E.	3.19		2.77	1.58	
2	S. 37¼° W.	5.86		4.66		3.55
4	N. 53° E.	19.32	11.63		15.43	
6	S. 60¾° W.	7.12		3.48		6.21
7	S. 29¼° E.	2.18		1.90	1.07	
8	S. 60½° W.	8.12		4.00		7.07
			11.63	16.81	18.08	16.83
				11.63	16.83	
Lat. and Dep. of closing line				5.18	1.25	

As lat. clos. line	5.18	A. C. 9.285670
: Dep.	1.25	0.096910
: : rad.		10.000000
: tan. bear.	N. 13° 34′ W.	9.382580

As cos. bear.		A. C. 0.012290
: rad.		10.000000
: : Lat.		0.714330
: dist. clos. line	5.33	0.726620

Clos. line	AB	5.33		
	BC	16.26	A. C.	8.788879
	AC	11.29	" "	8.947306
		2)32.88		
		16.44		1.215902
		11.11		1.045714
			2)	19.997801
½ ACB		4° 4′.		9.998900
ACB		8° 8′.		

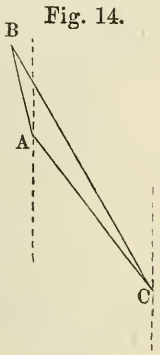


Fig. 14.

And

As AB	5.33	A. C. 9.273380
: AC	11.29	1.052694
:: sin. ACB		9.150686
: sin. ABC	17° 26′	9.476760

Bear. of BC = 17° 26′ + 13° 34′ = 31° 0′ S.E. 5th side.
Bear. of CA = 31° + 8° 8′ = 39° 8′ N.W. 3d side.

Article 357.

Ex. 2. Here ABC = 37° 15′.

As rad.		A. C. 0.000000
: sin. ABC	37° 15′	9.781966
:: AB	17.25	1.236789
:: BC	10.87	1.036230
: ABC	113.497	2.054985
ABC	56.7485 ch. = 5 A., 2 R., 27.98 P.	

Ex. 3.

As rad.		A. C. 0.000000
: sin. A	126° 47′	9.903581
:: AB	23.56	1.372175
:: AC	16.42	1.215373
: 2 ABC	309.834	2.491129
ABC	154.917 ch. = 15 A., 1 R., 38.67 P.	

Article 358.

Ex. 2.

As rad.		A. C. 0.000000
As sin. C	42° 21′	" " 0.171561
: sin. A	63° 52′	9.953166
: sin. B	73° 47′	9.982367
:: AB	17.63	1.246252
:: AB	"	1.246252
: 2 ABC	397.7385	2.599598
ABC	198.8692 ch. = 19 A., 3 R., 21.9 P.	

Ex. 3.

As	rad.		A. C. 0.000000
	sin.	57° 31′	0.073890
:	sin.	63° 17′	9.950968
	sin.	59° 12′	9.933973
::		15.65	1.194514
		"	1.194514
:	double area	222.771	2.347859
Area		111.3855 ch. = 11 A., 0 R., 22.17 P.	

Article 359.

Ex. 2. Here ABC = 93° and BCD = 106°.

As	rad.		A. C. 0.000000
:	sin. ABC	93°	9.999404
::	AB	12.47	1.095866
	BC	11.43	1.058046
:	1st quantity	142.336	2.153316

As	rad.		A. C. 0.000000
:	sin. C	106°	9.982842
::	BC		1.058046
	CD	9.16	0.961985
:	2d quantity	100.664	2.002873

As	rad.		A. C. 0.000000
:	sin. B + C − 180°	19°	9.512642
::	AB		1.095866
	CD		0.961985
:	3d quantity	37.196	1.570493
		100.664	
		142.336	
2 ABCD	=	280.196	
ABCD	=	140.098 ch. = 14 A., 0 R., 1.56 P.	

Ex. 3. Here B = 48° 30′ and C = 120° 30′.

As rad.		A. C. 0.000000
: sin. B	48° 30′	9.874456
:: AB	8.63	0.936011
:: BC	9.27	0.967080
: 1st quantity	59.917	1.777547

As rad.		A. C. 0.000000
: sin. C	120° 30′	9.935320
:: BC		0.967080
:: CD	11.23	1.050380
: 2d quantity	89.697	1.952780

As rad.		A. C. 0.000000
: sin. 180° − (B + C)	11°	9.280599
:: AB		0.936011
:: CD		1.050380
: 3d quantity	18.492	1.266990

$$\text{Area} = \frac{59.917 + 89.697 - 18.492}{2} = \frac{131.122}{2} = 65.566 \text{ ch.}$$

$$= 6 \text{ A., } 2 \text{ R., } 9 \text{ P.}$$

Article 361.

Ex. 3.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. 27° 15′ E.	7.75	6.89		3.55		0	0	6.89		3.55			10.91	00.00 E.		
2	S. 62° 25′ E.	10.80		5.00	9.57		0	1		5.00	9.56		13.11		13.11 E.		65.5500
3	S. 7° 55′ E.	9.50		9.41	1.31		0	1		9.41	1.30		10.86		23.97 E.		225.5577
4	S. 47° 25′ E.	9.37		6.34	6.90		0	0		6.34	6.90		8.20		32.17 E.		203.9578
5	S. 54° 25′ W.	8.42		4.90		6.85	0	0		4.90		6.85	.05		32.22 E.		157.8780
6	N. 37° 35′ W.	23.69	18.77			14.45	1	1	18.76			14.46		21.31	10.91 E.	204.6716	
			25.66	25.65	21.33	21.30						32.22	32.22				652.9435
																	204.6716
																	2) 448.2719
																	22 A., 1 R., 26.17 P. = 224.13595

Ex. 4.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. E.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. 46° 40′ W.	18.41	12.63			13.39	1	1	12.64			13.38		26.42			
2	N. 54° 30′ E.	13.45	7.81		10.95		0	0	7.81		10.95			2.43	2.43 W.		18.9783
3	N. 5° 30′ W.	12.65	12.59			1.21	0	0	12.59			1.21	9.74		7.31 E.	92.0329	
4	S. 74° 55′ E.	17.58		4.57	16.97		1	0		4.56	16.97		15.76		23.07 E.		105.1992
5	S. 47° 50′ E.	15.86		10.65	11.76		0	0		10.65	11.76		28.73		51.80 E.		551.6700
6	S. 47° 25′ W.	16.36		11.07		12.05	0	0		11.07		12.05		.29	51.51 E.		570.2157
7	S. 62° 35′ W.	14.69		6.76		13.04	0	0		6.76		13.04		25.09	26.42 E.		178.5992
				33.03	33.05	39.68	39.69										1424.6624
																	92.0329
																	2)1332.6295
																	66 A., 2 R., 21 P. = 666.31475

Ex. 5.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. 43° 25′ W.	28.43	20.65			19.54	0	2	20.65			19.56		46.29			
2	N. 29° 48′ E.	30.55	26.51		15.18		0	2	26.51		15.16			4.40	4.40 W.	116.6440	
3	S. 80° E.	28.74		4.99	28.30		0	2		4.99	28.28		43.44		39.04 E.		194.8096
4	N. 89° 55′ E.	40.00	.06		40.00		1	2	.07		39.98		68.26		107.30 E.	7.5110	
5	S. 10° 13′ E.	23.70		23.32	4.20		0	1		23.32	4.19		44.17		151.47 E.		3532.2804
6	S. 63° 55′ W.	25.18		11.07		22.62	0	2		11.07		22.64		18.45	133.02 E.		1472.5314
7	N. 63° 45′ W.	20.82	9.21			18.67	0	1	9.21			18.68		41.32	91.70 E.	844.5570	
8	S. 57° 25′ W.	31.70		17.07		26.71	1	2		17.06		26.73		45.41	46.29 E.		789.7074
			56.43	56.45	87.68	87.54			56.44	56.44	87.61	87.61				968.7120	5989.3288

968.7120

2)5020.6168

251 A., 0 R., 5 P. = 2510.3084

Ex. 6.

Sta.	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	S. 64° 5′ W.	S. 28° 45′ W.	11.18		9.80		5.38		13.40			
2	N. 49° 45′ W.	N. 85° 5′ W.	12.91	1.11			12.86		18.24	18.24 W.		20.2464
3	N. 35° 20′ E.	North.	(12.13)	(12.13)					12.86	31.10 W.		377.2430
4	S. 82° 25′ E.	N. 62° 15′ E.	(9.71)	(4.52)		(8.59)		8.59		22.51 W.		101.7452
5	N. 87° E.	N. 51° 40′ E.	13.82	8.57		10.84		19.43		3.08 W.		26.3956
6	N. 49° 30′ E.	N. 14° 10′ E.	4.95	4.80		1.21		12.05		8.97 E.	43.0560	
7	S. 33° 25′ E.	S. 68° 45′ E.	10.80		3.91	10.07		11.28		20.25 E.		79.1775
8	S. 55′ E.	S. 36° 15′ E.	9.22		7.44	5.45		15.52		35.77 E.		266.1288
9	S. 79° 10′ W.	S. 43° 50′ W.	14.30		10.32		9.90		4.45	31.32 E.		323.2224
10	N. 52° 15′ W.	N. 87° 35′ W.	8.03	.34			8.02		17.92	13.40 E.	4.5560	
				31.47	31.47	36.16	36.16				47.6120	1194.1589
												47.6120
												2)1146.5469
												A. 57,3.27345
												4
												R. 1.309380
												40
												P. 12.375200

As sin. bear.	62° 15′	A. C. 0.053063	As rad.		A. C. 0.000000		
: rad.		10.000000	: cos. bear.		9.668027		
: : dep.	8.59	0.933993	: : dist.		0.987056		
: dist. 4th side	9.71	0.987056	: lat.	4.52	0.655083		

Ex. 7.

Sta.	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	S. 45° E.	S. 43¼° E.	13.65		9.94	9.35		29.61				
2	N. 38¾° E.	N. 40½° E.	17.28	13.14		11.22		20.57		20.57 E.	270.2898	
3	N. 19° W.	N. 17¼° W.	23.43	22.38			6.95	4.27		24.84 E.	555.9192	
4	S. 58° W.	S. 59¾° W.	14.00		7.05		12.09		19.04	5.80 E.		40.8900
5	N. 87° W.	N. 85¼° W.	8.14	.67			8.11		20.20	14.40 W.		9.6480
6	N. 45½° W.	N. 43¾° W.	9.23	6.67			6.38		14.49	28.89 W.		192.6963
7	S. 28¼° W.	S. 30° W.	14.60		12.64		7.30		13.68	42.57 W.	538.0848	
8	S. 1¾° E.	South.			(16.44)				.7.30	49.87 W.	819.8628	
9	N. 79¼° E.	N. 81° E.		(3.21)		(20.26)		20.26		29.61 W.		95.0481
				46.07	46.07	40.83	40.83				2184.1566	338.2824
												2184.1566
												2)1845.8742
												A. 92,2.9371
												4
												R. 1.17484
												40
												P. 6.99360

As sin. bearing	81°	A.C. 0.005380	As rad.		0.000000	
: rad.		10.000000	: cos. bear.		9.194332	
: : dep.	20.26	1.306639	: : dist.		1.312019	
: dist.	20.51	1.312019	: lat.	3.21	1.506351	

Article 362.

Ex. 2.

52

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	S. 77¼° E.	11.32		2.50	11.04		1	1		2.49	11.03		18.43				
2	S. 6¾° E.	17.40		17.28	2.05		2	1		17.26	2.04		13.07		13.07 E.		225.5882
3	S. 72¾° W.	13.96		4.14		13.33	1	1		4.13		13.34		11.30	1.77 E.		7.3101
4	N. 32° W.	13.44	11.40			7.12	1	1	11.41			7.13		20.47	18.70 E.		213.3670
5	N. 30¾° E.	14.50	12.46		7.41		1	1	12.47		7.40		.27		18.43 E.		229.8221
			23.86	23.92	20.50	20.45											676.0874

Double area of offsets 18.9917

2) 695.0791

A. 34.753905

4

R. 3.015620

40

P. 0.624800

Additive. Inter. Dist.	Sum of Offsets.	Areas.
4.80	.75	3.6000
2.59	1.72	4.4548
1.57	1.69	2.6533
1.58	1.27	2.0066
.78	.55	.4290
1.01	.50	.5050
1.72	.95	1.6340
1.92	1.32	2.5344
2.21	1.22	2.6962
1.77	.98	1.7346
3.26	.63	2.0538

24.3017

Subtractive.		
2.95	.75	2.2125
1.57	1.50	2.3550
.99	.75	.7425

5.3100

18.9917

Ex. 3.

Sta.	Bearings.	D.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	S. 53° 10′ W.	11.28		6.76		9.03	1	1		6.75		9.04		2.54			
2	N. 29° 47′ W.	21.65	18.79			10.75	1	1	18.80			10.76		19.80	19.80 W.		372.2400
3	N. 67° 58′ E.	1.96	.74		1.82		0	0	.74		1.82			8.94	28.74 W.		21.2676
4	S. 30° 48′ E.	1.05		.90	.54		0	0		.90	.54		2.36		26.38 W.	23.7420	
5	S. 61° E.	9.89		4.79	8.65		0	1		4.79	8.64		9.18		17.20 W.	82.3880	
6	S. 73° 14′ E.	2.40		.70	2.30		0	0		.70	2.30		10.94		6.26 W.	4.3820	
7	S. 45° 28° E.	9.12		6.40	6.50		0	0		6.40	6.50		8.80		2.54 E.		16.2560
			19.53	19.55	19.81	19.78										110.5120	409.7636
																	110.5120
																	299.2516
															Double area of offsets		12.1812
																	2)311.4328
																	A. 15,5.7164
																	4
																	R. 2.28656
																	40
																	P. 11.46240

Inter. Dist.	Sum of Offsets.	Areas.
.11	.44	.0484
.94	.79	.7426
1.08	.75	.8100
1.92	1.72	3.3024
2.50	1.62	4.0500
4.39	.62	2.7218
1.50	.32	.4800
.40	.14	.0560
	.06	12.2112
Less .50		300
		12.1812

Article 363.

Ex. 2. Fig. 164. Surveying.

Sta.	Bearings.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	S. 56° 37′ W.	21.00		11.56		17.54	1			11.55		17.54		12.73			
2	S. 41° 7′ W.	10.90	8.21			7.17			8.21			7.17		24.71	24.71 W.		202.8691
3	N. 15° 15′ E.	8.42	8.12		2.21				8.12		2.21			4.96	29.67 W.		240.9204
4	N. 79° 7′ E.	18.01	3.40		17.69				3.40		17.69		19.90		9.77 W.		33.3157
5	S. 30° 24′ E.	9.50		8.19	4.81					8.19	4.51		22.50		12.73 E.		104.2587
			19.73	19.75	24.71	24.71											581.3639

Area of offsets 47.5159
Triangle 36.336 83.8519

2)497.5120

A. 24,8.7560

4

R. 3.5024

40

P. 20.0960

As rad. A. C. 0.000000
: sin FGH 44° 5′ 9.842424
:: { GF 6.27 0.797268
{ GH 8.33 0.920645
: 2 FGH 36.336 1.560337

Offsets.

Base.	Dist.	Offsets.	Inter. Dist.	Sum of Offsets.	Areas.
	.00	.00			
	.20	.15	.20	.15	.0300
	1.85	.25	1.65	.40	.6600
	3.50	.10	1.65	.35	.5775
	5.85	.82	2.35	.92	2.1620
AB	6.27		.42	.82	.3444
	14.60				
	15.00	.60	.40	.60	.2400
	16.60	.25	1.60	.85	1.3600
	18.90	.05	2.30	.30	.6900
	20.35	.55	1.45	.60	.8700
	21.00		.65	.55	.3575
	.00	.00			
	.45	.95	.45	.95	.4275
	1.50	.15	1.05	1.10	1.1550
	2.75	.10	1.25	.25	.3125
BC	4.75	.85	2.00	.95	1.9000
	6.75	.70	2.00	1.55	3.1000
	9.37	.05	2.62	.75	1.9650
	10.50	.50	1.13	.55	.6215
	10.90		.40	.50	.2000
	.00	.00			
	.15	.57	.15	.57	.0855
	2.25	.40	2.10	.97	2.0370
CD	4.00	.00	1.75	.40	.7000
	6.50	.37	2.50	.37	.9250
	7.85	.90	1.35	1.27	1.7145
	8.42		.57	.90	.5130
					22.9479

Base.	Dist.	Offsets.	Inter. Dist.	Sum of Offsets.	Areas.
	.00	.00			
	.55	.92	.55	.92	.5060
	2.75	.25	2.20	1.17	2.5740
	4.50	.05	1.75	.30	.5250
	6.50	.20	2.00	.25	.5000
DE	9.00	.10	2.50	.30	.7500
	11.50	.50	2.50	.60	1.5000
	13.50	.50	2.00	1.00	2.0000
	16.00	.10	2.50	.60	1.5000
	17.50	.50	1.50	.60	.9000
	18.01		.51	.50	.2550
	.00	.00			
	.27	.70	.27	.70	.1890
	1.50	.25	1.23	.95	1.1685
	3.00	.30	1.50	.55	.8250
EA	4.00	.00	1.00	.30	.3000
	5.00	.00	1.00	.00	.0000
	7.50	.70	2.50	.70	1.7500
	9.00	.20	1.50	.90	1.3500
	9.50		.50	.20	.1000
	.00	.00			
	.20	.90	.20	.90	.1800
GF	2.50	.00	2.30	.90	2.0700
	4.75	.65	2.25	.65	1.4625
	6.27		1.52	.65	.9880
	.00	.00			
	1.50	.20	1.50	.20	.3000
FH	3.00	.15	1.50	.35	.5250
	5.60	.70	2.60	.85	2.2100
	5.80		.20	.70	.1400
					24.5680
		Brought over,			22.9479
					47.5159

CHAPTER VII.

LAYING OUT AND DIVIDING LAND.

Article 378.

Ex. 2.

As	AB	11.25	A. C. 8.948847
	sin. A	73° 25′	" " 0.018451
:	2 ABC	70 ch.	1.845098
: :	rad.		10.000000
:	AC	6.49	0.812396

Ex. 3. Here B = 117° 15′; ∴

As	AB	16.37	A. C. 8.785951
	sin. B	117° 15′	" " 0.051090
:	ABCD	160	2.204120
: :	rad.		10.000000
:	BD	10.99	1.041161

Ex. 4. Here the given angle is 72°; ∴

As	14.37		A. C. 8.842543
	sin. 72°		" " 0.021794
:	double area	200	2.301030
: :	rad.		10.000000
:	other side	14.63	1.165367

Article 380.

Ex. 2. Here the angles are 102° 45′, 45° 45′, and 31° 30′.

∴ As	sin.	102° 45′	A. C. 0.010843
	sin.	45° 45′	" " 0.144904
:	rad.		10.000000
	sin.	31° 30′	9.718085
:: 2 area		100	2.000000
: square of side			2) 1.873832
Side		8.648	.936916

Ex. 3. Here A = 62° 16′, B = 48°, C = 69° 44′.

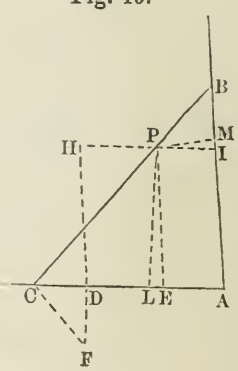
Then

As	sin. A	62° 16′	A. C. 0.052996
	sin. B	48	" " 0.128927
:	rad.		10.000000
	sin. C	69° 44′	9.972245
:: 2 ABC		240	2.380211
: AB^2			2) 2.534379
AB		18.50	1.267189

Article 381.

$$\text{Ex. 2.} \quad AD = \frac{60}{7.92} = 7.5757.$$

Fig. 15.



As sin. I	85°	A. C.	0.001656
: sin. M	90		10.000000
:: PM	3.25		0.511883
: PI	3.26		0.513539

$PH = AD - IP = 4.32.$

$DC = \sqrt{FC^2 - FD^2} = \sqrt{7.58 \times 1.06}$

$= \sqrt{8.0348} = 2.84.$

$AC = 7.58 + 2.84 = 10.42.$

Ex. 3. Fig. 172. Surveying.

As sin. I	88° 15′	A. C. 0.000203
: sin. PAI	42°	9.825511
:: PA	5.64	0.751279
: PI	3.776	0.576993
As rad.		A. C. 0.000000
: sin. PAL	46° 15′	9.858756
:: PA		0.751279
: PL	4.074	0.610035
Given area	35	1.544068
FE + FD = AD	8.591	.934033
FE − FD	1.039	.016616
		2) .950649
DC	2.99	.475324
AD	8.59	
AC =	11.58	

Article 383.

Ex. 2. Fig. 175. Surveying.

As sin. A	90°	A. C. 0.000000
As sin. B	71° 30′	" " 0.023043
: rad.		10.000000
: sin. C	18° 30′	9.501476
:: 2 ABCD	140	2.146128
: $AB^2 - CD^2$	46.8432	1.670647
AB^2 =	161.7984	

CD = $\sqrt{114.9552} = 10.72$.

And

As sin. E	18° 30′	A. C. 0.498524
: sin. B		9.976957
:: AB − CD	2.00	0.301030
: AD	5.98	0.776511

Ex. 3. Fig. 174. Surveying.

As	sin. A	90°	A. C. 0.000000
	sin. B	119° 30′	" " 0.060303
:	rad.		10.000000
	sin. E	29° 30′	9.692339
::	2 ABCD	240	2.380211
:	$CD^2 - AB^2$	135.785	2.132853

$AB^2 = 134.56$

$CD = \sqrt{270.345} = 16.44.$

As	sin. E		A. C. 0.307661
:	sin. B		9.939697
::	CD − AB	4.84	0.684845
:	AD	8.555	0.932203

Article 384.

Ex. 2. Fig. 176. Surveying. Here A = 91° and B = 87° 30′. Hence K lies below F, and AL is greater than AI.

Area	100 ch.	2.000000
AB	7.86	A. C. 9.104577
AI	12.72	1.104577
AI	12.72	1.104577
tan. KAI	2° 30′	8.640093
IK	.555	−1.744670
And AI		1.104577
tan IAF	1°	8.241921
IF	.222	−1.346498

Whence FK = IK − IF = .333.

And EF = AB − FK = 7.527.

Then

As 2 EF	15.054	A. C. 8.822348
: AI	12.72	1.104577
:: FK	.333	−1.522444
: IL	.281	−1.449369

AL = AI − IL = 13.001.

Article 385.

Ex. 2. Fig. 178. Surveying.

As sin. E	18° 30′	A. C. 0.498524
: sin. B	71° 30′	9.976957
:: AB	12.72	1.104487
: AE	38.016	1.579968
AB	12.72	1.104487
sin. A	90°	10.000000
2 ABE	483.565	2.684455
2 ABCD	140	
2 CDE	343.565	

As { sin. E		A. C. 0.498524
{ sin. D	116° 47′	" " 0.049286
: { rad.		10.000000
{ sin. C	81° 43′	9.995446
:: 2 ECD	343.565	2.536009
: ED^2		2) 3.079265
ED	34.644	1.539632
AE	38.016	
AD	3.372	

And

As sin. C	81° 43′	A. C. 0.004554
: sin. E	18° 30′	9.501476
:: ED		1.539632
: CD	11.11	1.045662

Ex. 3. Here A = 106° 56′, B = 111° 4′, E = 38°, C and D each 71°, and bear. of CD N. 30° 43′ E.

As sin. E	38°	A. C. 0.210658
: sin. B	111° 4′	9.969957
:: AB	15.62	1.193681
: AE	23.675	1.374296
AB		1.193681
sin. A	106° 56′	9.980750
2 ABE	353.775	2.548727
2 ABCD	200	
2 CDE	553.775	

And since EC = ED, we shall have

As sin. E		A. C. 0.210658
: 2 ECD	553.775	2.743333
:: rad.		10.000000
: ED^2		2) 2.953991
ED	29.991	1.476995
EA	23.675	
AD	6.316	

Article 386.

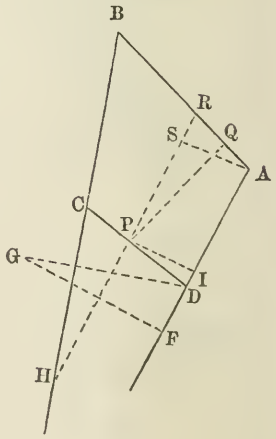
Ex. 2. Produce AF and BC to meet in E.

To find AB, ABE, and EF.

As sin. E	11°	A. C. 0.719401
: sin. B	71° 16′	9.976361
:: AB	15.23	1.182700
: AE	75.59	1.878462
AB		1.182700
sin. A	97° 44′	9.996032
2 ABE	1140.76	3.057194
ABE	570.38	
ABCD	150	
ECD	420.38	

ECD	420.38	2.623642
PI	9.22	0.964731
EF	45.59	1.658911

Fig. 16.



To find PR, AR, and BR.

As sin. R	82° 16′	A. C. 0.003968
: rad.		10.000000
:: PQ	10.55	1.023252
: PR	10.647	1.027220

As sin. R	82° 16′	A. C. 0.003968
: rad.		10.000000
:: AS	9.22	0.964731
: AR	9.305	0.968699
AB	15.23	
BR	5.925	

To find HR and HP.

As sin. E	11°	A. C. 0.719401
: sin. B	71° 16′	9.976361
:: BR	5.925	0.772668
: HR	29.407	1.468450
PR	10.647	
HP	18.76	

DG + FG = EF =	45.59	1.658911
DG − FG = EF − 2 HP =	8.07	0.906874
		2) 2.565785
FD	19.18	1.282892

Whence AD = EA − (EF + FD) = 10.82.

Article 387.

Ex. 1. Fig. 180. Surveying.

To find AE, ABE, and AD.

As sin. E	30° 30′	A. C. 0.294531
: sin. B	44° 10′	9.843076
:: AB	16.75	1.224015
: AE	22.99	1.361622
AB		1.224015
sin. A	105° 20′	9.984259
2 ABE	371.446	2.569896
2 ABCD	300	
Then 2 ECD	671.446	

And

As sin. E		A. C. 0.294531
: rad.		10.000000
:: 2 ECD		2.827011
: ED^2		2) 3.121542
ED	36.37	1.560771
AE	22.99	
AD	13.38	

Bear. of DC, N. $11\frac{1}{4}$ W.

Article 388.

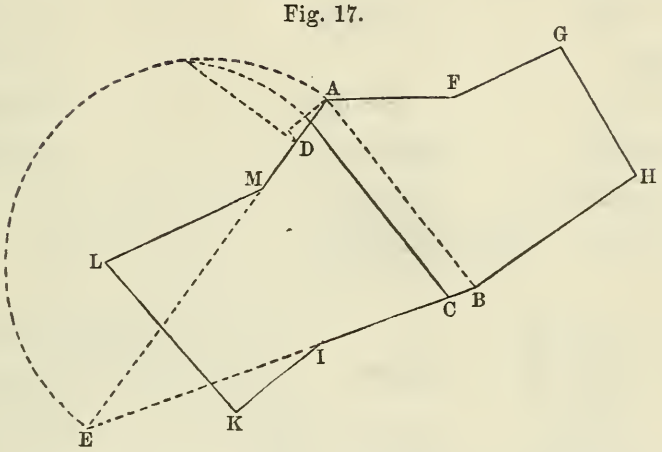
St.	Bearings.	Ch. Bear.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Mult.	S. Areas.
1	N. $27\frac{1}{4}$° W.	N. $1\frac{1}{4}$° E.	5.00	5.00		.11			15.53	15.53 W.	77.6500
2	N. 58° W.	N. $29\frac{1}{2}$° W.	9.53	8.29			4.69		4.58	20.11 W.	166.7119
3	N. $42\frac{1}{2}$° E.	N. 71° E.	9.60	3.13		9.08		4.39		15.72 W.	49.2036
4	S. $81\frac{1}{4}$° E.	S. $52\frac{3}{4}$° E.	14.00		8.47	11.14		20.22		4.50 E.	38.1150
5	S. $28\frac{1}{2}$° E.	South.	(10.76)		(10.76)			11.14		15.64 E.	
							(15.64)		15.64	00.00 E.	
						20.33					331.6805
											500
											168.3195

$$\text{Lat. of 5th side} = \frac{168.3195}{15.64} = 10.76.$$

Article 389.

Ex. 2.

Fig. 17.



	N.	S.	E.	W.	E.D.D.	W.D.D.	Mult.	N. Areas.	S. Areas.
AF		.32	14.83			1.45			
FG	5.10		12.31		27.14		27.14 E.	138.4140	
GH		14.82	7.63		19.94		47.08 E.		697.7256
HB		11.78		18.49		10.86	36.22 E.		426.6716
BA	(21.82)			(16.28)		34.77	1.45 E.	31.6390	
	26.92	26.92	34.77	34.77				170.0530	1124.3972
									170.0530
							2 AFGHB		954.3442
							2 given area		1100.
							2 ABCD		145.6558

As Lat. AB 21.82 A. C. 8.661145
: Dep. 16.28 1.211654
:: rad. 10.000000
: tan. bearing 36° 44′ 9.872799

As cos. bearing A. C. 0.096131
: rad. 1.338855
:: Lat. 10.000000
: Dist. AB 27.23 1.434986

Produce AM and BI to meet in E. Then

As sin. E	34°	A. C. 0.252438
: sin. B	70° 16′	9.973716
:: AB		1.434986
: AE	45.839	1.661140
AB		1.434986
sin. A	75° 44′	9.986395
2 ABE	1209.27	3.082521
2 ABCD	145.66	
2 CDE	1063.61	

As { sin. E		A. C. 0.252438
{ sin. D	104°	" " 0.015056
: { rad.		10.000000
{ sin. C	110°	9.970152
:: 2 CDE	1063.61	3.026782
: DE^2		2) 3.264428
DE	42.876	1.632214
EA	45.839	
DA	2.963	

MD = MA − DA = 9.207 ch.

Article 391.

Ex. 2. Fig. 182. Surveying. The division line will fall on the fourth side.

Sta.	Bearings.	Ch. Bear.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Mult.	N. Areas.	S. Areas.
10	N. 22½° E.	N. 11° W.	9.00	8.83			1.72		37.81	37.81 W.		333.8623
11	S. 76¾° E.	N. 69¾° E.	12.00	4.15		11.26		9.54		28.27 W.		117.3205
12	S. 15° W.	S. 18½° E.	10.85		10.29	3.44		14.70		13.57 W.	139.6353	
13	S. 18° W.	S. 15½° E.	10.62		10.23	2.84		6.28		7.29 W.	74.5767	
1	N. 75° E.	N. 41½° E.	13.70	10.26		9.08		11.92		4.63 E.	47.5038	
2	N. 20½° E.	N. 13° W.	10.30	10.03			2.32	6.76		11.39 E.	114.2417	
3	East.	N. 56½° E.	16.20	8.94		13.51		11.19		22.58 E.	201.8652	
4	S. 33½° W.	South.			(3.51)			13.51		36.09 E.		
D.					(18.18)		(36.09)		36.09			
				42.21	42.21	40.13	40.13				577.8227	451.1828
											451.1828	

As diff. lat. div. line 18.18 A. C. 8.740406
: Dep. " 36.09 1.557387
:: rad. 10.000000
: tan. bearing 63° 16′ 10.297793
33 30
96 46
180
Div. line N. 83° 14′ W.

cos. bearing 63° 16′ A. C. 0.346943
Lat. 18.18 1.259594
Dist. 40.41 1.606537

36.09) 126.6399 (3.51
108.27
18.369
18.045
.3249

Article 399.

Ex. 2.

As ABC	58 ch.	A. C. 8.236572
: ADE	30 "	0.477121
:: AB	12.25	1.088136
:: AC	10.42	1.017868
: AD . AE		1.819697
AE	8.50	0.929419
AD	7.77	.890278

Article 400.

Ex. 2.

As $5 : 2 :: AC^2\ (400) : AD^2 = 160$,
$\therefore AD = \surd 160 = 12.65$.

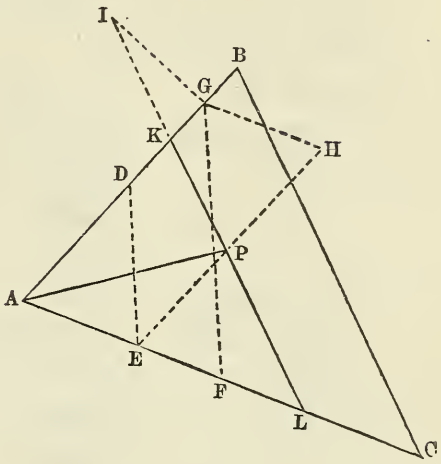
Article 401.

Here, CF being perpendicular to AB, we have

As rad.		A. C. 0.000000
: cos. A.	39° 45′	9.885837
:: AC	13.24	1.121888
: AF		1.007725
AG	6.365	.803798
		2)1.811523
AD	8.049	.905761

Article 402.

Fig. 18.



Ex. 2. To find AE and EP.

As sin. AEP	111° 30′	A. C. 0.031322
: sin. APE	33° 45′	9.744739
: : AP	5.90	0.770852
: AE	3.523	0.546913

And

As sin. AEP		A. C. 0.031322
: sin. PAE	34° 45′	9.755872
: : AP		0.770852
: PE	3.614	0.558046

To find AG.

As AE		A. C. 9.453087
: AF	5.935	0.773421
: : AD	4.4	0.643453
: AG = KI + IG =	7.412	0.869961
2 EP	7.228	
KI − IG	.184	−1.264818
		2) 0.134779
KG =	1.168	0.067389

And AK = AG ± KG = 7.412 ± 1.168 = 6.244 or 8.58.

Article 403.

Ex. 2 Fig. 19.

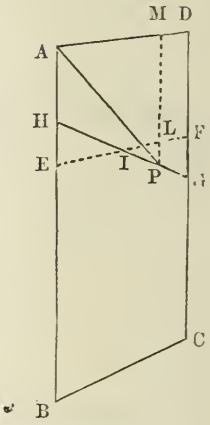
$DF = \frac{1}{3} CD = 3.96$ and $AE = \frac{1}{3} AB = 4.977$.

To find PM and AM.

As sin. M	83° 50′	A. C. 0.002520
: sin. PAM	54° 50′	9.912477
: : AP	6.20	0.792392
: PM	5.098	0.707389

As sin. M		A. C. 0.002520
: sin. APM	41° 20′	9.819832
: : AP		0.792392
: AM	4.119	0.614744

Fig. 1.



To find ML, LP, and FG.

As AD	5.18	A. C. 9.285670
: AM		0.614744
: : AE − DF	1.017	0.007321
: AE − ML	.809	−1.907735
AE	4.977	
ML	4.168	
LP = MP − ML =	.93	

As AM − ½ AD	1.529	A. C. 9.815593
: ½ AD	2.59	0.413300
: : PL	.93	−1.968483
: FG = EH	1.575	0.197376

whence

DG = DF + FG = 5.535, and AH = AE − EH = 3.402.

Article 404.

Ex. 2.

$$EF = \sqrt{\left(\frac{4BC^2 + 5 . AD^2}{9}\right)} = \sqrt{\left(\frac{426.0096 + 154.0125}{9}\right)}$$

$$= \sqrt{64.4469} = 8.03.$$

And as
BC − AD (4.77) : FE − AD (2.48) :: AB (6.47) : AE = 3.36.

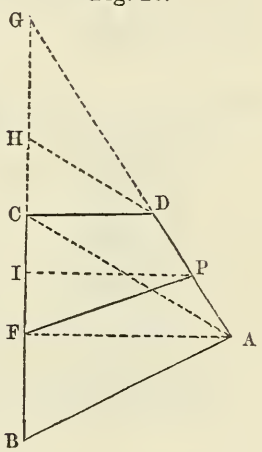
Article 405.

Ex. 2.

To find AG and BG.

As sin. G	34°	A. C. 0.252438
: sin. B	62° 30′	9.947929
:: AB	19.55	1.291147
: AG	31.01	1.491514
As sin. G		A. C. 0.252438
: sin. A	83° 30′	9.997199
:: AB		1.291147
: GB	34.74	1.540784

Fig. 20.



To find GH and BI.

As GA	31.01	A. C. 8.508486
: GD	18.76	1.273233
:: GC	15.82	1.199206
: GH	9.57	1.980925

BI = $\frac{5}{9}$(GB − GH) = 13.98; GI = GB − BI = 20.76.

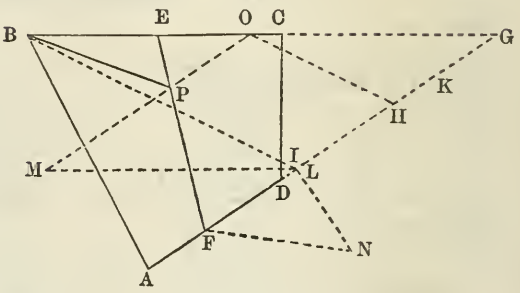
To find GF and BF.

As GP	25.01	A. C. 8.601886
: GA	31.01	1.491514
:: GI	20.76	1.317227
: GF	25.74	1.410627
GB	34.74	
BF	9.00	

Article 406.

Ex. 2.

Fig. 21.



As in Ex. 2 of last article, find AG = 31.01, GB = 34.74, GD = 18.76, GC = 15.82.

Hence,

As GB	34.74	A. C. 8.459216
: GC	15.82	1.199206
: : GD	18.76	1.273233
: GH	8.544	0.931655

AH = GA − GH = 22.466

AI = $\frac{5}{9}$ AH = 12.481, GI = 18.529, and GK = 9.264.

To find BO, OP, and GL.

As sin. BOP	34°	A. C. 0.252438
: sin. BPO	126° 15′	9.906575
: : BP	11.52	0.061452
: BO	16.614	1.220465

As sin. BOP		A. C. 0.252438
: sin. PBO	19° 45′	9.528810
: : BP		1.061452
: PO	6.961	0.842700

As GO	18.126	A. C. 8.741698
: GB	34.74	1.540784
: : GK	9.264	0.966799
: GL	17.753	1.249281

To find FL and AF.

FN + LN = LG	1.249281
FN − LN = LG − 2 PO = 3.831	0.583312
	2) 1.832593
FL = 8.247	.916296

AF = AG − (FL + LG) = 5.01.

Article 407.

Ex. 2.

As	sin. E	77°	A. C. 0.011276
	sin. F	128° 45′	" " 0.107970
:	sin. A	76°	9.986904
	sin. D	78° 15′	9.990803
: :	AD	8.35	0.921686
	AD	"	0.921686
:	Fourth term	87.162	1.940325
		2	
		174.324	
3 BC^2		119.07	
		5) 293.394	

$EF = \sqrt{58.6788} = 7.66.$

And

As sin. G	25° 45′	A. C. 0.362065
: sin. E		9.988724
: : EF − BC	1.36	0.133539
: BF	3.05	.484328

AF = AB − BF = 1.60.

Ex. 3.

As	sin. E	77°	A. C. 0.011276
	sin. F	78° 15′	" " 0.009197
:	sin. A	76°	9.986904
	sin. B	128° 45′	9.892030
: :	AB	4.65	0.667453
	AB	4.65	0.667453
:	xy^2	17.152	1.234313

$$EF = \sqrt{\left(\frac{3\,xy^2 + 2\,CD^2}{5}\right)} = \sqrt{\left(\frac{51.456 + 106.58}{5}\right)} = 5.62.$$

And

As sin. G	24° 45′	A. C. 0.378139
: sin. E	77°	9.988724
: : CD − EF	1.68	0.225309
: FA	3.91	0.592172

Article 408.

As { sin. E	65° 15′	A. C. 0.041846
{ sin. F	90°	" " 0.000000
: { sin. A	76°	9.986904
{ sin. B	128° 45′	9.892030
: : { AB	4.65	0.667453
{ AB	4.65	0.667453
: xy^2	18.017	1.255686

As { sin. E		A. C. 0.041846
{ sin. F		" " 0.000000
: { sin. C	77°	9.988724
{ sin. D	78° 15′	9.990803
: : { CD	7.30	0.863323
{ CD	7.30	0.863323
: vw^2	55.978	1.748019

$$EF = \sqrt{\left(\frac{3\,xy^2 + 2\,vw^2}{5}\right)} = \sqrt{33.2014} = 5.762.$$

As sin. E		A. C. 0.041846
: sin. B		9.892030
: : AB	4.65	0.667453
: BO	3.993	0.601329

And

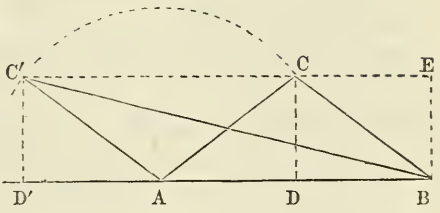
As sin. G	24° 45′	A. C. 0.378139
: sin. E		9.958154
: : EF − BO	1.769	0.247728
: AF	3.837	0.584021

CHAPTER VIII.

MISCELLANEOUS EXAMPLES.

Ex. 1. Make AB = 50, one of the given sides; erect the perpendicular $BE = \frac{2 \text{ Area}}{AB} =$ 19.2. With the circle A, and radius equal to the other given side, (32,) describe an arc cutting EC, a parallel through E, in C and C′: then will ABC or ABC′ be the triangle required.

Fig. 22.

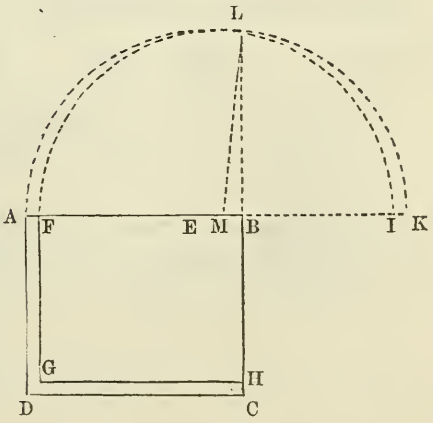


Calculation.—$AD = \sqrt{AC^2 - AD^2} = \sqrt{655.36} = 25.6$, and (12.13.2) $BC^2 = BA^2 + AC^2 \pm 2\,BA \,.\, AD = 2500 + 1024 \pm 2560 = 6084$, or 964;

whence $BC = 78$, or 31.05.

Ex. 2. Let ABCD be the garden. Produce AB, making $BI = \frac{1}{8}AB$. On AI describe the semicircle ALI, cutting CB produced in L. Make AE = AD, and bisect EB in M. With the centre M and radius ML describe a semicircle cutting AB in F and K. Make BH = BK: then will AF or CH be the breadth of the walk.

Fig. 23.



For FH = FB . BK = BL^2 = AB . BI = $\frac{7}{8}$ AB . BC = $\frac{7}{8}$ ABCD; also, MF = MK, and EM = MB; whence FE = BK = BH; ∴ AF = HC.

Calculation.—BI = $\frac{7}{8}$ BC = 87.5; BL^2 = AB . BI = 10500.

$$ML = \sqrt{MB^2 + BL^2} = \sqrt{10600} = 102.95.$$

$$AF = AE + EM - MF = 7.05.$$

Ex. 3. Similar triangles are as the squares of their homologous sides.

The area of the triangle, whose sides are 3, 4, and 5, is 6.

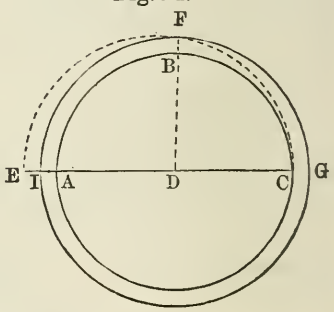
Whence 6 ch. : 24 ch. :: 16 : 64, and $\sqrt{64} = 8$, one side;

∴ 6 and 10 ch. are the others.

Ex. 4. *Construction.*—Let ABC be the plat. Produce the diameter CA to E, making AE = $\frac{1}{4}$ AD. On EC describe a semicircle cutting the perpendicular DB in F. DF will be the radius of the outer circle.

For, since circles are as the squares of their radii, we have

Fig. 24.



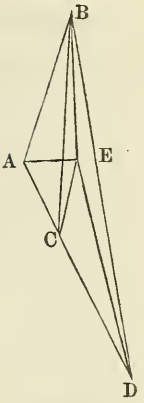
ABC : IFG :: DC^2 : DF^2. But CD : DF :: DF : DE;

whence CD^2 : DF^2 :: CD : DE :: 4 : 5;

∴ ABC : IFG :: 4 : 5.

Calculation.—4 : 5 :: AD^2 : DI^2 :: 5625 : 7031.25, and DI = 83.85; whence AI = 8.85.

Fig. 25.



Ex. 5. Let AB represent the tree, and C and D the two stations. Then we have ACE = 5° 37′, ADE = 2° 29′, BCE = 67° 43′ 30″, CD = 100, and CA = 75.

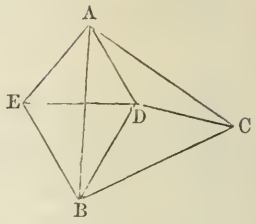
As sin. CED	3° 8′	A. C. 1.262332
: sin. CDE	2° 29′	8.636776
: : CD	100°	2.000000
: CE	79.27	1.899108
tan. BCE	67° 43′ 30″	10.387619
BE	193.52	2.286727
BE^2	37451	4.573454

As CE + CA	154.27	A. C. 7.811719
: CE − CA	4.27	0.630428
: : tan. $\frac{CAE + CEA}{2}$	87° 11′ 30″	11.309328
: tan. $\frac{CAE - CEA}{2}$	29° 26′ 3″	9.751475
CEA	57° 45′ 27″	

As sin. CEA	57° 45′ 27″	A. C. 0.072733
: sin. ACE	5° 37′	8.990660
: : CA	75	1.875061
: AE	8.679	0.938454
AE^2	75.32	1.876908
EB^2	37451	
AB =	$\sqrt{37526.32}$ = 193.7.	

Ex. 6. Describe the triangle ADE, having the sides ED, AD, and DE equal to 6.23, 4.95, and 5.62 respectively. On DE describe the equilateral triangle DBE. Join AB, and on it describe the equilateral triangle ABC.

Fig. 26.



This will be the one required.

For, since ABC = EBD, ABE = CBD; whence, in the two triangles ABE and CBD, we have two sides and the included angle of one equal to two sides and the included angle of the other; consequently AE = DC.

Calculation.

AD	4.95	
DE	6.23	A. C. 9.205512
AE	5.62	" " 9.250264
	2)16.80	
	8.40	0.924279
	3.45	0.537819
		2)19.917874
cos. $\frac{1}{2}$ AED	24° 31′ 29″	9.958937
AED	49° 2′ 58″	

As EB + EA	11.85	A. C. 8.926282
: EB − EA	.61	−1.785330
:: tan. $\frac{\text{EAB} + \text{EBA}}{2}$	35° 28′ 31″	9.852871
: tan. $\frac{\text{EAB} - \text{EBA}}{2}$	2° 6′ 3″	8.564483

Then

As sin. EBA	33° 22′ 28″	A.C. 0.259541
: sin. AEB	109° 2′ 58″	9.975540
:: EA	5.62	0.749736
: AB		0.984817

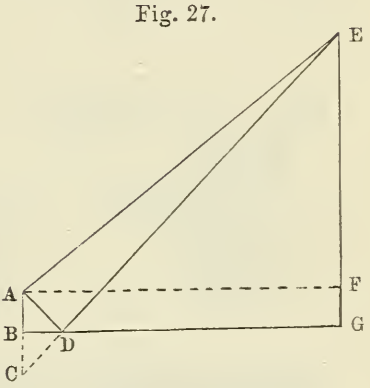
Finally,

As rad.		A. C. 0.000000
: sin. 60°		9.937531
:: AC		0.984817
:: AB		0.984817
: 2 ABC	80.754	1.907165

ABC = 40.377 ch. = 4 A., 0 R., 5.6 P.

Ex. 7.

Fig. 27.



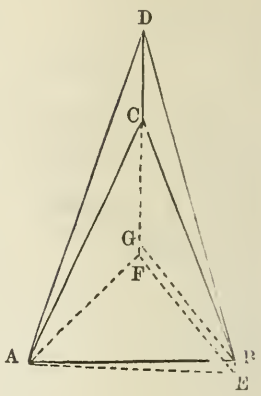
As sin. AEC	7° 43′	A. C. 0.872007
: sin. ACE	44° 34′	9.846175
:: AC	60	1.778151
: AE		2.496333

As rad.		A. C. 0.000000
: sin. EAF	37° 43′	9.786579
:: AE		2.496333
: EF	191.83	2.282912
	30	
Height	221.83	

And

As rad.		A. C. 0.000000
: cos. A		9.898201
:: AE		2.496333
: AF	248.05	2.394534

Fig. 28.



Ex. 8. Through A conceive the horizontal plane AEF to pass, and draw BG parallel to EF. Then we have FAE = 87° 45′, FAC = 39° 43′, FAD = 52° 13′, BAE = 2° 17′, AEF = 54° 23′, GBC = 33° 4′, and GBD = 45° 42′.

First. To find AE.

As rad.		A. C. 0.000000
: cos. BAE	2° 17′	9.999655
:: AB	157	2.195900
: AE	156.88	2.195555

To find AF, FC, and FD.

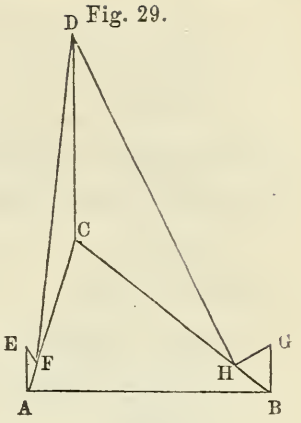
As sin. AFE	37° 52′	A. C. 0.211955
: sin. AEF	54° 23′	9.910054
:: AE		2.195555
: AF	208.54	2.317564
tan. CAF	39° 43′	9.919448
Elev. of hill = FC	172.5	2.237012

As rad.		A. C. 0.000000
: tan. FAD	52° 13′	10.110579
:: AF		2.317564
: FD	268	2.428143
FC	172.5	

Height of tower CD = 95.5

The calculation might have been made by finding BE = 6.25, EF = BG = 255.37, GC = 166.26, and GD = 261.69. Whence FC = 172.51, FD = 267.93, and CD = 95.43.

Fig. 29.



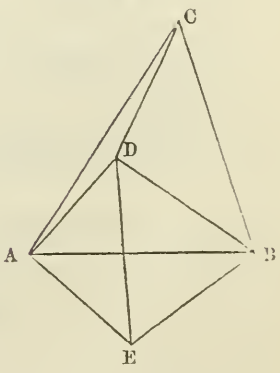
Ex. 9. It is readily seen that AC : BC :: tan. DHC : tan. DFC :: cot. DFC : cot. DHC. But AC : BC :: sin. ABC : sin. BAC. Whence

As cot. DHC	32°	A. C. −1.795789
: cot. DFC	39° 48′	10.079267
:: sin. BAC	90°	10.000000
: sin. ABC	48° 35′ 23″	9.875056

As sin. BCA	41° 24′ 37″	A. C. 0.179506
: sin. ABC		9.875056
:: AB	100	2.000000
: AC		2.054562
tan.	39° 48′	9.920733
	94.47	1.975295
	10	
CD =	84.47	

Ex. 10. Make BD = 9.43, BDE = 49° 50′, DBE = 72° 41′, DA = 7.56, and AE = $\frac{BE}{BD} \times 8.42$; also make ABC = EBD and BAC = BED: then will ABC be the triangle required.

Fig. 30.



For, since ABC and EBD by construction are similar, we have EB : BD :: AB : BC. But ABC = EBD; whence ABE = CBD. Consequently (6.6) ABE

and CBD are similar; whence BE : EA :: BD : DC, or $AE = \frac{BE}{BD} . DC$. But by construction $AE = \frac{BE}{BD} \times 8.42$. Whence DC = 8.42; and, since AD = 7.56 and BD = 9.43, the conditions of the problem are fulfilled.

To find ED.

As sin. BED	57° 29′	A. C. 0.074051
: sin. DBE	72° 41′	9.979855
:: DB	9.43	0.974512
: DE	10.6762	1.028418

To find AE we have by construction BD : BE :: DC : AE; or,

As sin. BED	57° 29′	A. C. 0.074051
: sin. BDE	49° 50′	9.883191
:: DC	8.42	0.925312
: AE	7.6305	0.882554

To find ADE.

AE	7.6305	
AD	7.56	A. C. 9.121478
DE	10.6762	8.971582
	2) 25.8667	
$\frac{1}{2}\ s$	12.93335	1.111711
$\frac{1}{2}\ s$ − AE	5.30285	0.724510
		2) 19.929281
cos. $\frac{1}{2}$ ADE	22° 48′ 29″	9.964640
ADE	45° 36′ 58″	

whence ADB = ADE + EDB = 95° 26′ 58″.

To find AB.

As BD + DA	16.99	A. C. 8.769807
: BD − DA	1.87	0.271842
:: tan. $\frac{BAD + ABD}{2}$	42° 16′ 31″	9.958631
: tan. $\frac{BAD - ABD}{2}$	5° 42′ 51″	9.000280
As sin. BAD	47° 59′ 22″	0.128998
: sin. BDA	95° 26′ 58″	9.998032
:: BD	9.43	0.974512
: BA	12.63	1.101542

To find BC and AC.

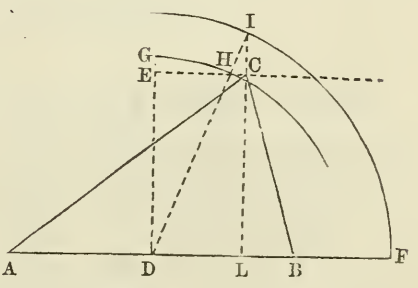
As sin. C	49° 50′	A. C. 0.116809
: sin. BAC	57° 29′	9.925949
:: AB		1.101542
: BC	13.94	1.144300

And

As sin. C		A. C. 0.116809
: sin. ABC	72° 41′	9.979855
:: AB		1.101542
: AC	15.78	1.198206

Ex. 11. Let AB = 50. Bisect it in D, and make DF = 42.5 the half sum of the two sides. At D erect the perpendicular DG. Make DE = 2 Area ÷ AB = 32. Lay off BG = DF. With D as a centre, describe the arcs FI and GH, the latter cutting EC parallel to AB in H. Join DH, and produce it to I. Draw ICL perpendicular to AB, cutting EC in C. Then will C be the vertex of the triangle.

Fig. 31.



It is evident, from the construction, that the base and

area agree with the data. It remains to be proved that AC + BC = 85.

Put AD = DB = a, DF = BG = b, DG = c, DL = d, and LC = e: then we have $a^2 + c^2 = b^2$; or $b^2 - c^2 = a^2$.

By similar triangles, $DI^2 : DH^2 :: IL^2 : LC^2$; or $b^2 : c^2 :: b^2 - d^2 : e^2$; whence $e^2 = \frac{b^2c^2 - c^2d^2}{b^2} = c^2 - \frac{c^2d^2}{b^2}$.

$AC^2 = AL^2 + LC^2 = (a + d)^2 + e^2 = a^2 + 2\,ad + d^2 + c^2 - \frac{c^2d^2}{b^2} = b^2 + 2\,ad + \frac{(b^2 - c^2)\,d^2}{b^2} = b^2 + 2\,ad + \frac{a^2d^2}{b^2}$;

whence $AC = b + \frac{ad}{b}$

Similarly $BC = b - \frac{ad}{b}$ $\therefore AC + BC = 2\,b = 85$.

Calculation.

$DG^2 = DF^2 - DB^2 = 1806.25 - 625 = 1181.25$.

Then, As $DH^2 : DI^2 :: DE^2 : IL^2$,

or As $1181.25 : 1806.25 :: 1024 : IL^2 = 1565.7989$;

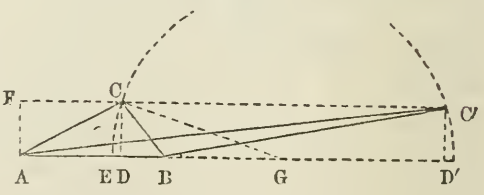
whence $DL = \sqrt{DI^2 - IL^2} = 15.5065$,

and $AC = \sqrt{AL^2 + LC^2} = \sqrt{1640.77654225 + 1024} = 51.6215$;

$\therefore BC = 85 - AC = 33.3785$.

Ex. 12. Make AB = 20 the given side, and at A erect the perpendicular AF = 2 ABC ÷ AB = 7. Divide AB in E, so that AE : EB :: 3 : 2. Take BG a third proportional to AE − EB and EB. With the centre G and radius GE describe a circle cutting FC parallel to AB in C and C′. Then will ABC or ABC′ be the triangle required.

Fig. 32.



For, since $AE - EB : EB :: EB : BG$,
by composition, $AE : EB :: EG : BG$.
Alternately, $AE : EG :: EB : BG$;
whence, by composition, $AG : EG :: EG : BG$.
and (F.6) $AC : CB$, or $A'C : C'B :: AE : EB :: 3 : 2$.

Calculation.

$AE = 12$, $BE = 8$, $BG = 16$, $GE = 24$, $DC = 7$;

whence $GD = GD' = \sqrt{GC^2 - CD^2} = \sqrt{527} = 22.96$.

Then (12.13.2) $BC^2 = BG^2 + GC^2 \pm 2\,BG \,.\, GD = 256 + 576 \pm 734.72 = 97.28$, or 1566.72.

Whence $BC = 9.86$ or 39.58,

and $AC = \frac{3}{2} BC = 14.79$ or 59.37.

Ex. 13.

Sta.	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. $83\frac{1}{2}°$ E.	N. 69° E.	2.37	.85		2.21			13.06	13.06 W.		11.1010
2	S. 47° E.	S. $61\frac{1}{2}°$ E.	6.25		2.98	5.49		7.70		5.36 W.	15.9728	
3	N. $62\frac{3}{4}°$ E.	N. $48\frac{1}{4}°$ E.	5.17	3.44		3.86		9.35		3.99 E.	13.7256	
4	S. $56\frac{1}{2}°$ E.	S. 71° E.	3.92		1.28	3.71		7.57		11.56 E.		14.7968
5	N. $14\frac{1}{2}°$ E.	North.			(.25)			3.71		15.27 E.		
				(.22)			(15.27)		15.27	00.00		
				4.51	4.51						29.6984	25.8978
											25.8978	

As diff. lat. .22 A. C. 10.657577
: dep. 15.27 1.183839
: : rad. 10.000000
: tan. bear. N. 89° 10′ W. 11.841416
14° 30′
N. 74° 40′ W. reverse bearing of division line.

15.27) 3.8006 (.25
3.054
7466
7635

Ex. 14.

	Bearings.	Dist.	N.	S.	E.	W.
1	S. 26½° E.	1.53		1.37	.68	
2	N. 60½° E.	19.37	9.54		16.86	
3	N. 28½° W.	3.27	2.87			1.56
				(11.04)		(15.98)
			12.41	12.41	17.54	17.54

As diff. lat.	11.04	A. C. 8.957031
: dep.	15.98	1.203577
: : rad.		10.000000
: tan. bear.	N. 55° 22′ E.	10.160608

As cos. bear.	55° 22′	A. C. 0.245405
: rad.		10.000000
: : diff. lat.		1.042969
: dist.	19.43	1.288374

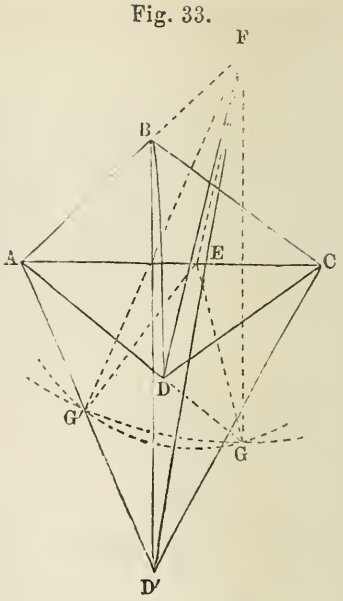
	Bearings.	Dist.	N.	S.	E.	W.
	S. 55° 22′ W.	8.53		4.85		7.02
	S. 26½° E.	1.53		1.37	.68	
	N. 60½° E.	8.53	4.20		7.42	
			(2.02)			(1.08)
			6.22	6.22	8.10	8.10

As diff. lat.	2.02	A. C. 0.694649
: dep.	1.08	0.033424
: : rad.		10.000000
: tan. bear.	N. 28° 8′ W.	9.728073

As cos. bear.		A. C. 0.054604
: rad.		10.000000
: : lat.		0.305351
: dist.	2.29	.359955

Ex. 15. A slight investigation will show that the distances from the observers to the point directly under the balloon are as the cotangents of the angles of elevation. These cotangents are, $a = .73412$, $b = .94345$, and $c = .75996$. Hence the following construction:—

Fig. 33.



Let A, B, and C be the places of the observers. Take AE = AB and AF = AC. At the points E and F, with radii equal to $\frac{c}{a}$. AE and $\frac{b}{a}$. AF, describe arcs cutting in G: join AG and make ACD = AGE. Then will D be the point under the balloon.

Since the two arcs will cut in G′ as well as G, there will be two points (D and D′) that will answer the conditions.

Because AGE = ACD, the triangles ADC and AEG are similar; whence AD : DC :: AE : EG :: a : c; and AG : AE :: AC : AD ∴ AG : AC (AF) :: AE . (AB) : AD. Since, therefore, AFG and ADB have the common angle at A and the containing sides proportional, they are similar, and consequently AD : DB :: AF : FG :: a : b.

Calculation.

To find FG and EG.

As a	.73412	A. C. 10.134230
: c	.75996	−1.880790
:: AE	1000	3.000000
: EG	1035.19	3.015020

As a		A. C. 10.134230
: b	.94345	−1.974719
:: AF	.1690	3.227887
: FG	2171.88	3.336836

To find AEF and FEG.

AF	1690	
AE	1000	A. C. 7.000000
EF	1180	" " 6.928118
	2)3870	
	1935	3.286681
$\frac{1}{2}s - AF$	245	2.389166
		2)19.603965
cos $\frac{1}{2}$ AEF	50° 39′ 57″	9.801982
AEF	101° 19′ 54″	

FG	2171.88	
FE	1180	A. C. 6.928118
EG	1035.19	" " 6.984980
	2)4387.07	
	2193.535	3.341145
$\frac{1}{2}s - FG$	21.655	1.335558
		2)18.589801
cos. $\frac{1}{2}$ FEG	78° 37′ 37″	9.294900
FEG	157° 15′ 14″	

AEG = 360° − (AEF + FEG) = 101° 24′ 52″ = ADC.

To find DAC and DCA.

As AD (a)		A. C. 10.134230
: DC (c)		−1.880790
:: rad.		10.000000
: tan. x	45° 59′ 26″	10.015020

As rad.		A. C. 0.000000
: tan. $x - 45°$	59′ 26″	8.237799
:: tan. $\frac{DAC + ACD}{2}$	39° 17′ 34″	9.912902
: tan. $\frac{DAC - ACD}{2}$	48′ 38″	8.150701
DAC	40° 6′ 12″	

As sin. ADC	101° 24′ 52″	A. C. 0.008677
: sin. DAC	40° 6′ 12″	9.808999
:: AC	1690	3.227887
: CD	1110.6	3.045563
tan.	52° 46′	10.119210
Height	1461.4	3.164773

or AEG′ = FEG − AEF = 55° 55′ 20″ = AD′C.

Then, as AD′ : D′C :: rad. : tan. x. = 45° 59′ 26″ as before.

And

As rad.		A. C. 0.000000
: tan. $x - 45°$	59′ 26″	8.237799
:: tan. $\frac{D'AC + ACD'}{2}$	62° 2′ 10″	10.275037
: tan. $\frac{D'AC - ACD'}{2}$	1° 52′ 0″	8.512836
D′AC	63° 54′ 20″	

As sin. AD′C	55° 55′ 20″	A. C. 0.081825
: sin. D′AC	63° 54′ 20″	9.953311
:: AC	1690	3.227887
: CD	1832	3.263023
tan.	52° 46′	10.119210
Height	2411	3.382233

Ex. 16.

To find the area of the whole tract.

Sta.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	S. Areas.
1	10.82			19.78		57.71		
2	15.81			2.79		22.57	22.57 W.	356.8317
3	6.99		12.48		9.69		12.88 W.	90.0312
4		6.75	15.64		28.12		15.24 E.	102.8700
5		1.56	22.35		37.99		53.23 E.	83.0388
6		16.26	10.03		32.38		85.61 E.	1392.0186
7		9.05		37.93		27.90	57.71 E.	522.2755
	33.62	33.62	60.50	60.50				2) 2547.0658
								1273.5329

To find the area to a line north from the first station.

Sta.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	S. Areas.
1	10.82			19.78		19.78		
2	15.81			2.79		22.57	22.57 W.	356.8317
3	6.99		12.48		9.69		12.88 W.	90.0312
4		4.35	(10.09)		22.57		9.69 E.	42.1515
5		(29.27)			10.09		19.78 E.	578.9606
	33.62	33.62	22.57	22.57				1067.9750

As rad.			A. C. 0.000000
: cot. bear. 4th side	S. 66° 40′ E.		9.634838
: : Dep.	10.09		1.003891
: Lat.	4.35 S.		0.638729

1273.5329 — 1067.9750 = 205.5579 = double area between the division line and a line due north 29.27 ch. from the first station.

Then

As { sin.	76° 35′		A. C. 0.012017
{ sin.	66° 40′		" " 0.037055
: { rad.			10.000000
{ sin.	36° 45′		9.776937
: : 2 area	205.5579		2.312934
: fourth term	137.703		2.138943
$(29.27)^2$ =	856.7329		

$\sqrt{719.0299} = 26.815$ = division line.

As sin.	36° 45′	A. C. 0.223063
: sin.	66° 40′	9.962945
: :	2.455	0.390051
: Dist. from 1st cor.	3.77	0.576059

Ex. 17. As it is of no importance to the calculation of this question what the course of the line is, we shall sup-

pose it to be north. The bearings of the different lines in course will then be as below.

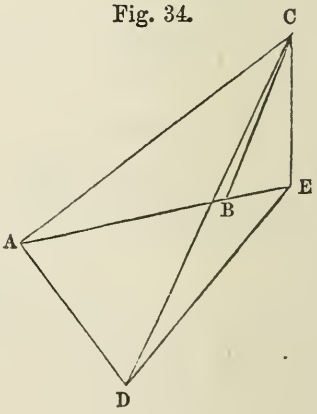
Sta.	Bearings.	Dist.	N.	S.	E.	W.
1	South.			(23.67)		
2	N. 67° 35′ E.	10.00	3.81		9.24	
3	N. 18° 52′ E.	7.25	6.86		2.34	
4	N. 7° 7′ E.	5.43	5.39		.67	
5	N. 58° 10′ W.	(14.42)	(7.61)			(12.25)

As rad.		A. C. 0.000000
: cot. bear. 5th side	58° 10′	9.792974
:: Dep.	12.25	1.088136
: Lat.	7.61	0.881110

As sin. bear.		A. C. 0.070793
: rad.		10.000000
:: Dep.		1.088136
: Dist. 5th side	14.42	1.158929

Distance of line, 23.67.

Fig. 34.



Ex. 18. Here we have AE : CE :: cot. CAE : rad., and DE : EC :: cot. CDE : rad. ∴ AE : ED :: cot. CAE : cot. CDE. Whence cot. CDE : cot. CAE :: sin. DAE : sin. ADE.

As cot. CDE	20° 4′	A. C. 9.562636
: cot. CAE	25° 10′	10.328037
:: sin. DAE	90°	10.000000
: sin. ADE	51° 1′ 40″	9.890673

As rad.		A. C. 0.000000
: tan. ADE		10.092061
: : AD	60	1.778151
: AE	74.17	1.870212

As rad.		A. C. 0.000000
tan. CAE	25° 10′	9.671963
: : AE		1.870212
: EC		1.542175

As EC		A. C. 8.457825
: EB	4.17	0.620136
: : rad.		10.000000
: cot. EBC	83° 10′ 34″	9.077961

As cos. EBC		A. C. 0.925118
: rad.		10.000000
: : BE		0.620136
: BC	35.1 yds.	1.545254

Ex. 19.

Sta.	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E.D.D.	W.D.D.	Multipliers.	N. Areas.	S. Areas.
1	N. 61° 20′ W.	N. 24° 40′ E.	22.55	20.49		9.41			17.77	17.77 W.		364.1073
2	N. 10° W.	N. 76° E.	16.05	3.88		15.57		24.98		7.21 E.	27.9748	
3	N. 60° 45′ E.	S. 33° 15′ E.	14.30		11.96	7.84		23.41		30.62 E.		366.2152
4	S. 66° 40′ E.	S. 19° 20′ W.	17.03		16.07		5.64	2.20		32.82 E.		527.4174
5	S. 86° E.	South.			(1.61)				5.64	27.18 E.		
6				(5.27)			(27.18)		27.18	00.00		
				29.64	29.64	32.82	32.82					1257.7399
												27.9748
												1229.7651
										Double area (see Ex. 16)		1273.5329

As lat. 5.27 A. C. 9.278189
: dep. 27.18 1.434249
:: rad. 10.000000
: tan. bear. N. 79° 1′ W. 10.712438
S. 86° E.
165° 1°
180°
S. 14° 59′ W.

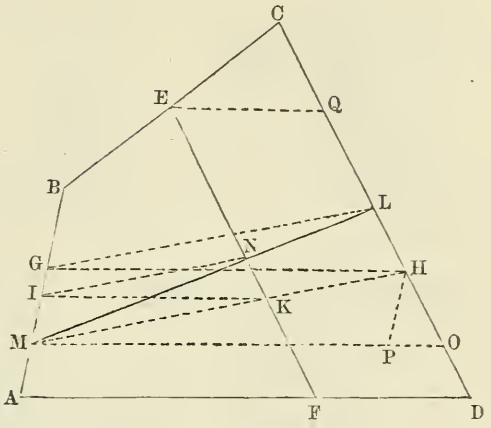
As cos. bear. 79° 1′ A. C. 0.720052
: rad. 10.000000
:: lat. 0.721811
: dist. 27.66 1.441863

27.18) 43.7678 (1.61
27.18
16.587
16.308
.2798

Hence the bearing of the division line is N. 14° 59′ E. 27.66 chains.

Ex. 20. Let ABCD be the quadrilateral. Divide it into two equal parts by the line EF parallel to CD. (Art. 407.) Then divide the quadrilaterals ABCD and ABEF into two equal parts by the lines GH and IK parallel to AD. Join HK, and produce it to M. Through G draw GL parallel to MH. Then will ML be the other division line.

Fig. 35.



For, since GL is parallel to MH, the triangles MGH and MLH are equal: to each add AMHD, and we have AMLD = AGHD = $\frac{1}{2}$ ABCD.

Again, since NK is parallel to LH and IN to GL, we have MN : NL :: MK : KH :: MI : IG.

Whence IN is parallel to GL. The triangle MIK is therefore equal to MNK: to each add AMKF, and we have AMNF = AIKF = $\frac{1}{4}$ ABCD.

To find AF and EF.

As	sin. C	78° 45′	A. C. 0.008426
	sin. D	63°	" " 0.050119
:	sin. A	78° 30′	9.991193
	sin. B	139° 45′	9.810316
: :	AB	23	1.361728
	AB		1.361728
:	fourth term	383.274	2.583510
CD^2		2161.3201	
		2)2544.5941	
EF		$= \sqrt{1272.2970} = 35.67.$	

As sin.	38° 15′	A. C. 0.208243
: sin. C	78° 45′	9.991574
:: CD — EF	10.82	1.034227
: FD	17.14	1.234044
AD	49.64	
AF	32.50	

To find AG and GH.

As { sin. A		A. C. 0.008807
{ sin. D		" " 0.050119
: { sin. B		9.810316
{ sin. C		9.991574
:: { BC	30.50	1.484300
{ BC		1.484300
: fourth term	675.15	2.829416
AD^2	2464.1296	
	2)3139.2796	

GH = $\sqrt{1569.6398}$ = 39.62.

As sin.	38° 30′	A. C. 0.205850
: sin. D		9.949881
:: AD — GH	10.02	1.000868
: AG	14.34	1.156599

Draw EQ parallel to AD. Then

As sin. CEQ	38° 15′	A. C. 0.208243
: sin. CQE		9.949881
:: CQ	10.82	1.034227
: EC	15.57	1.192351
BC	30.50	
BE	14.93	

To find AI and IK.

As	sin. A		A. C. 0.008807
	sin. F		" " 0.050119
:	sin. B		9.810316
	sin. E		9.991574
::	BE	14.93	1.174060
	BE		1.174060
:	fourth term	161.79	2.208936

$AF^2 = 1056.25$

$2)1218.04$

$IK = \sqrt{609.02} = 24.68.$

As sin.	38° 30′	A. C. 0.205850
: sin. F		9.949881
:: AF − IK	7.82	0.893207
: AI	11.19	1.048938

Now, because IK is parallel to GH, we have
GH : IK :: MG : MI; whence, by division,
GH − IK : IK :: IG : MI; or
14.94 : 24.68 :: 3.15 : MI = 5.20;
and AM = AI − IM = 5.99.

Through M and H draw MO and HP parallel to AD and AB. Then in HPO we have

As sin. HOP (D)		A. C. 0.050119
: sin. HPO (A)		9.991193
:: HP	8.35	0.921686
: HO	9.18	0.962998

As sin. HOP		A. C. 0.050119
: sin. PHO	38° 30′	9.794150
:: HP		0.921686
: HO	5.83	0.765955

And by similar triangles MOH and GHL we have MO (45.45) : OH (9.18) :: GH (39.62) : HL = 8.

Then in MOL we have MO, OL, and MOL, to find OML. Thus

As MO + OL	62.63	8.203218
: MO − OL	28.27	1.451326
:: tan. $\frac{\text{MLO} + \text{OML}}{2}$	58° 30′	10.212681
: tan. $\frac{\text{MLO} - \text{OML}}{2}$	36° 23′	9.867225
OML	22° 7′	
Bearing of MO	S. 66° 15′ E.	
" " ML	S. 88° 22′ E.	

98

THE END.

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